CAMOSUN COLLEGE

School of Access

Academic and Career Foundations Department

MATH 057 unit 6

Vectors

<u>MATH 057 unit 6 Vectors</u>

- MATH 057 unit 6 is for students who plan to enter the Electrical ELT program.
- Unit 6 applies right triangle trigonometry to solve problems involving vectors.

Definitions

Many quantities can be described completely by giving only their size or magnitude. Such quantities are called **scalar** quantities.

Examples of scalar quantities are *length* or *distance* (1.3 m), *speed* (80 mph), *area* (240 cm^2), *volume* (500 ml), *mass* (2.5 kg), and *temperature* (20 °C).

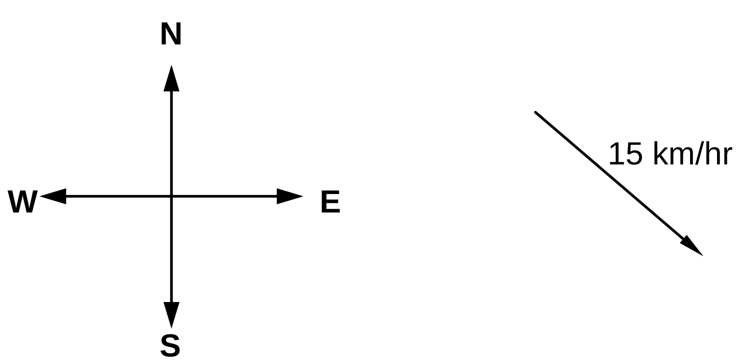
Other quantities can only be described completely by giving both their magnitude and direction. Such quantities are called **vector** quantities or **vectors**.

Examples of vector quantities, which specify both size and direction, are *displacement* (45 km west), *velocity* (50 km/hr south), and *force* (780 newtons downwards).

In the study of AC circuits, *voltage* (60 volts @ 30°), *current* (100 amps @ 120°), and *impedance* (20 ohms @ 60°) are also vectors, since they specify both magnitude and direction or “phase angle”.

Vectors are represented graphically by directed line segments, or “arrows”.
The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector.

For example, using a scale of 1 cm per 5 km/hr, a 15 km/hr wind from the northwest would be represented by an arrow 3 cm long, as shown below.



Addition of Perpendicular Vectors

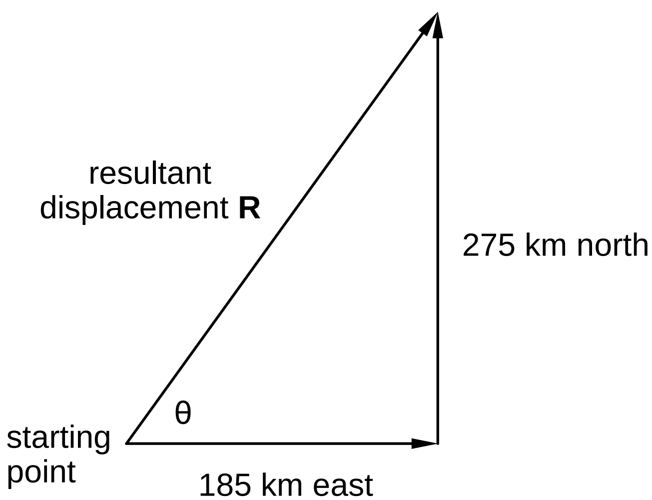
Vector “addition” is the process of combining vectors into a **resultant** vector or “vector sum”, which is a single vector that has the same effect as the original vectors.

Example 1 A plane flies due east for 185 km, makes a 90° turn, and then flies due north for 275 km. Find the plane’s resultant displacement from its starting point (both distance and direction).

We sketch the two vectors “head to tail” as shown below, and draw the resultant displacement **R** by connecting the tail of the first vector to the head of the second.

The length of **R** represents the distance from the starting point, and the angle θ represents the direction.

This method of adding vectors is called the **vector triangle method**.



Since this is a right triangle, we use the Pythagorean Theorem to find the distance **R**:

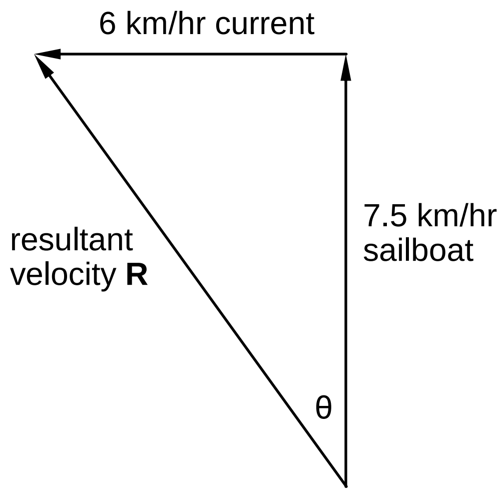
$$\mathbf{R}^2 = 185^2 + 275^2$$
$$\mathbf{R} = \sqrt{185^2 + 275^2} = 331.4 \text{ km}$$

We use right triangle trigonometry to find the angle θ between **R** and the 185 km vector:

$$\theta = \tan^{-1} \frac{275}{185} = 56.1°$$

The plane’s resultant displacement is 331.4 km from its starting point, in the direction 56.1° north of east. This could also be described as 331.4 km @ 56.1°, if we express the direction as an angle in **standard position** (counterclockwise from the positive *x* axis).

Example 2 A boat sails due north at 7.5 km/hr, but it is pushed off course by a 6 km/hr current from the east. Find the resultant velocity of the sailboat (both rate of speed and direction).



Using the Pythagorean Theorem to find the actual rate of speed **R**:

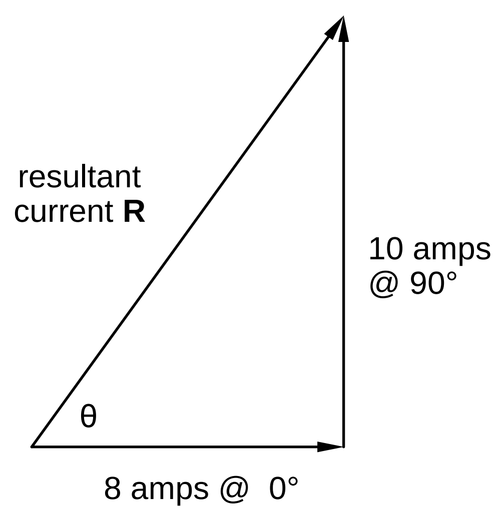
$\mathbf{R} = \sqrt{7.5^2 + 6^2} = 9.6$ km/hr

Using trigonometry to find the angle θ between **R** and the intended direction north:

$\theta = \tan^{-1} \frac{6}{7.5} = 38.7°$

The sailboat's resultant velocity is 9.6 km/hr, in the direction 38.7° west of north (or 9.6 km/hr @ 128.7°, expressing the angle in standard position).

Example 3 Determine the resultant line current in a parallel AC circuit, when the two branch currents are 8 amps @ 0° and 10 amps at 90°.



Using the Pythagorean Theorem to find **R**:

$\mathbf{R} = \sqrt{8^2 + 10^2} = 12.8$ amps

Using trigonometry to find the phase angle θ:

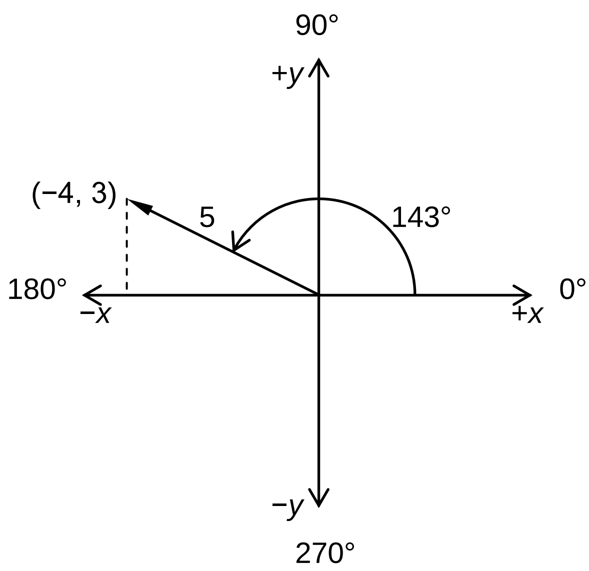
$\theta = \tan^{-1} \frac{10}{8} = 51.3°$

The resultant line current is 12.8 amps @ 51.3°.

Components of Vectors

Finding the **components** of a given vector is the reverse of vector addition.
That is, instead of combining two perpendicular vectors into one resultant vector,
we can separate or **resolve** any given vector into its horizontal and vertical components.

As shown below, a vector may be given in **polar form** (5 @ 143°), which describes its magnitude and direction as an angle in standard position (counterclockwise from the positive *x* axis), or in **rectangular form** (−4, 3), which describes the horizontal and vertical coordinates of the head of the vector (like the *x*-coordinate and *y*-coordinate of a point).



Example 4 A ship travels 375 kilometres in a direction 27.3° south of east.
Find the east (horizontal) and south (vertical) components of its displacement **D**.

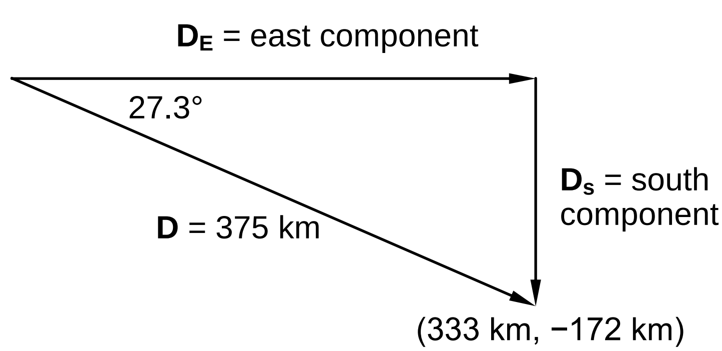
Using trigonometry to find the east and south components:

$\cos 27.3° = \frac{D_E}{375}$

$\mathbf{D_E} = 375 \cdot \cos 27.3° = 333$ km

$\sin 27.3° = \frac{D_S}{375}$

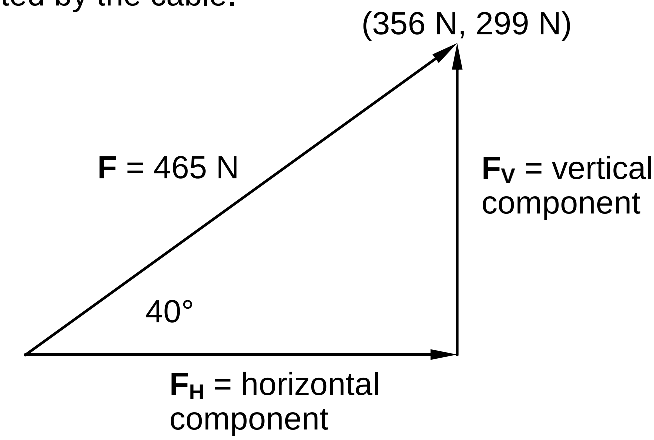
$\mathbf{D_S} = 375 \cdot \sin 27.3° = 172$ km



The ship has traveled 333 km east and 172 km south.

The ship's displacement may be written in polar form as 375 km @ 332.7° (360° − 27.3°),
or in rectangular form as (333 km E, 172 km S) or (333 km, −172 km).

Example 5 A cable exerts a force of 465 newtons (N) at an angle of 40° with the horizontal. Resolve this force into its horizontal component and its vertical component, which is the maximum weight that can be lifted by the cable.



$\cos 40° = \frac{F_H}{465}$

$\mathbf{F_H} = 465 \cdot \cos 40° = 356$ N

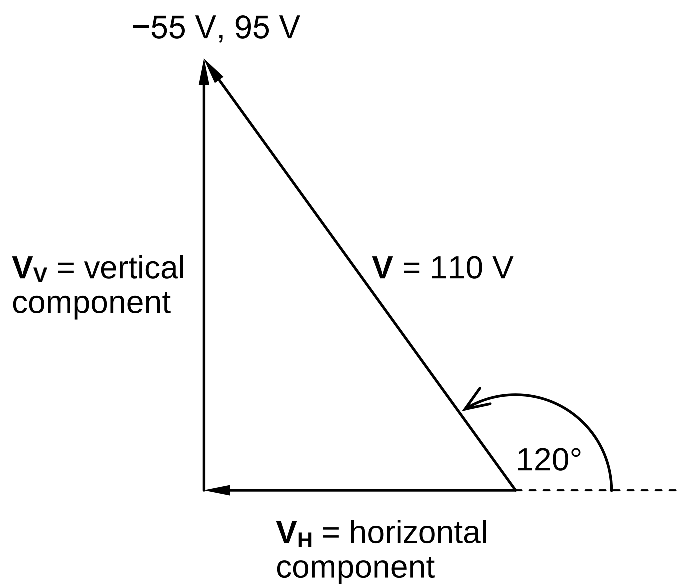
$\sin 40° = \frac{F_V}{465}$

$\mathbf{F_V} = 465 \cdot \sin 40° = 299$ N

The cable exerts a horizontal force of 356 N and a vertical force of 299 N.

The pulling force of the cable may be written in polar form as 465 N @ 40°, or in rectangular form as (356 N, 299 N).

Example 6 Find the horizontal and vertical components of a voltage of 110 V @ 120°.



$\cos 120° = \frac{V_H}{110}$

$\mathbf{V_H} = 110 \cdot \cos 120° = -55$ V

$\sin 120° = \frac{V_V}{110}$

$\mathbf{V_V} = 110 \cdot \sin 120° = 95$ V

The voltage is written in polar form as 110 V @ 120°, or in rectangular form as (−55 V, 95 V).

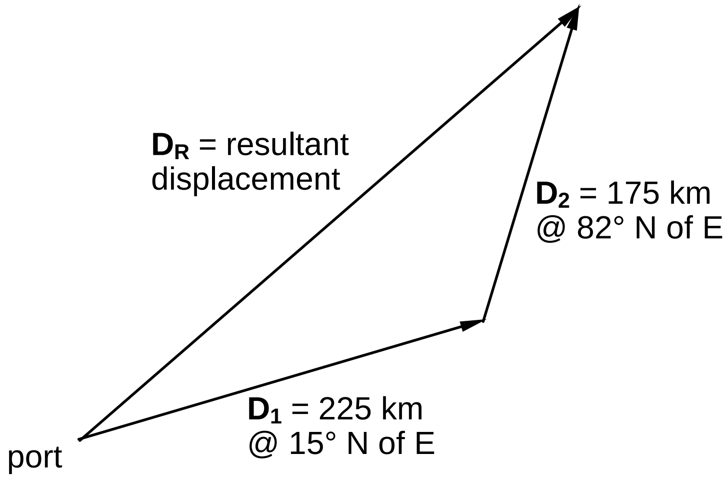
<u>To find the components of a given vector in polar form</u>

- for the **horizontal component**, multiply the magnitude of the vector by the **cosine** of the vector's angle in standard position
- for the **vertical component**, multiply the magnitude of the vector by the **sine** of the vector's angle in standard position

Addition of Vectors by Using Components

We can now add any number of vectors in any position, by adding their horizontal and vertical components.

Example 7 A ship leaves port and sails 225 km in a direction 15° north of east, then turns and sails 175 km in a direction 82° north of east. Find the resultant displacement of the ship (both distance and direction).



As shown in the table below, we

- express the direction of each vector as an angle in standard position
- find the horizontal and vertical components of each vector
- add the horizontal components and add the vertical components of both vectors

vector in polar form	horizontal component	vertical component
$\mathbf{D_1}$ = 225 km @ 15°	$225 \cdot \cos 15° = 217.3$ km	$225 \cdot \sin 15° = 58.2$ km
$\mathbf{D_2}$ = 175 km @ 82°	$175 \cdot \cos 82° = 24.4$ km	$175 \cdot \sin 82° = 173.3$ km
total	241.7 km	231.5 km

To find the resultant displacement $\mathbf{D_R}$ of the ship, we combine the totals of the horizontal and vertical components into one resultant vector.

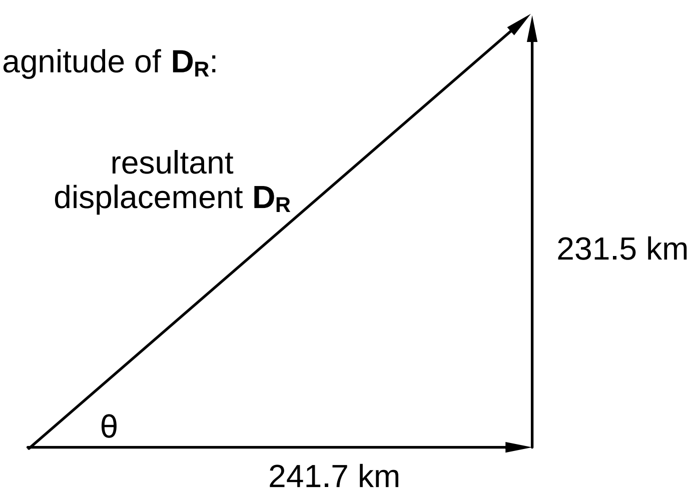
Using the Pythagorean Theorem to find the magnitude of $\mathbf{D_R}$:

$$\mathbf{D_R} = \sqrt{241.7^2 + 231.5^2} = 334.7 \text{ km}$$

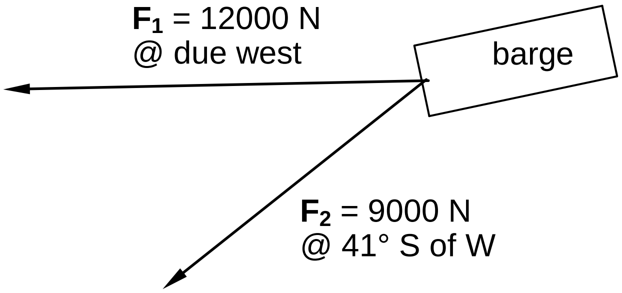
Using trigonometry to find the direction of $\mathbf{D_R}$:

$$\theta = \tan^{-1} \frac{231.5}{241.7} = 43.8°$$

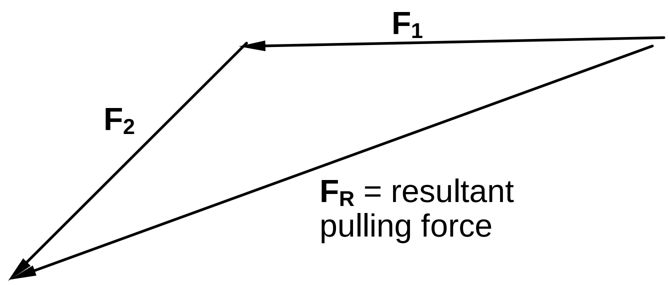
The resultant displacement of the ship is 334.7 km @ 43.8° (standard position).



Example 8 Two tugboats are pulling a barge. The largest tug exerts a pulling force of 12000 newtons due west, while the smaller tug exerts a force of 9000 newtons in a direction 41° S of W. Find the resultant pulling force of both tugs (both magnitude and direction).



head to tail sketch



vector in polar form	horizontal component	vertical component
$\mathbf{F_1}$ = 12000 N @ 180°	12000 · cos 180° = −12000 N	12000 · sin 180° = 0 N
$\mathbf{F_2}$ = 9000 N @ 221°	9000 · cos 221° = −6792 N	9000 · sin 221° = −5905 N
total	−18792 N	−5905 N

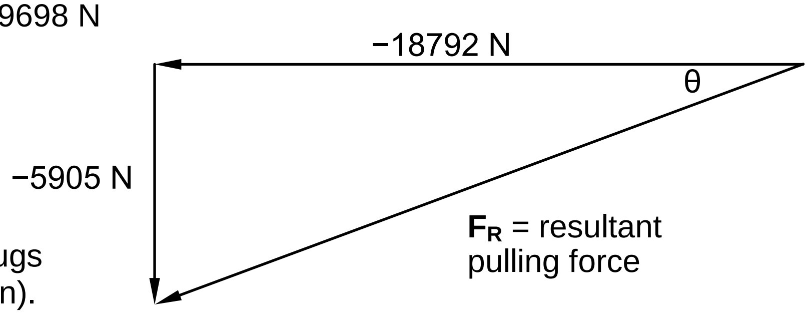
Combining the totals of the horizontal and vertical components into one resultant vector, we use the Pythagorean Theorem to find the magnitude of $\mathbf{F_R}$:

$$\mathbf{F_R} = \sqrt{(-18792)^2 + (-5905)^2} = 19698 \text{ N}$$

To find the direction of $\mathbf{F_R}$:

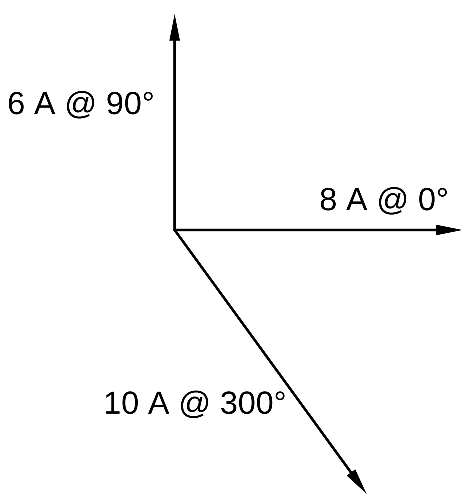
$$\theta = \tan^{-1} \frac{-5905}{-18792} = 17.4°$$

The resultant pulling force of the two tugs is 19698 N @ 197.4° (standard position).



Example 9

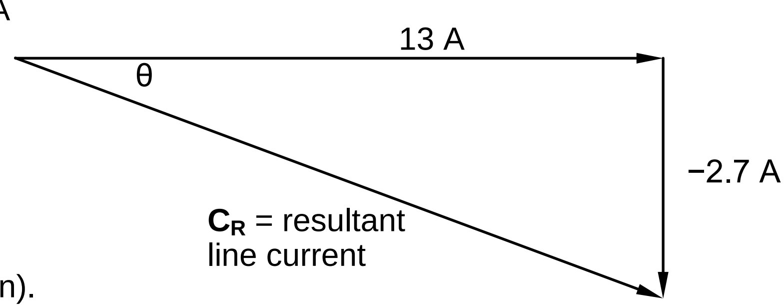
Find the resultant line current in a parallel AC circuit when the three branch currents are 8 A at 0°, 6 A at 90°, and 10 A at 300°.



vector in polar form	horizontal component	vertical component
8 A @ 0°	8 · cos 0° = 8 A	8 · sin 0° = 0 A
6 A @ 90°	6 · cos 90° = 0 A	6 · sin 90° = 6 A
10 A @ 300°	10 · cos 300° = 5 A	10 · sin 300° = −8.7 A
total	13 A	−2.7 A

Combining the horizontal and vertical totals into one resultant current vector $\mathbf{C_R}$:

$$\mathbf{C_R} = \sqrt{(13)^2 + (-2.7)^2} = 13.3 \text{ A}$$



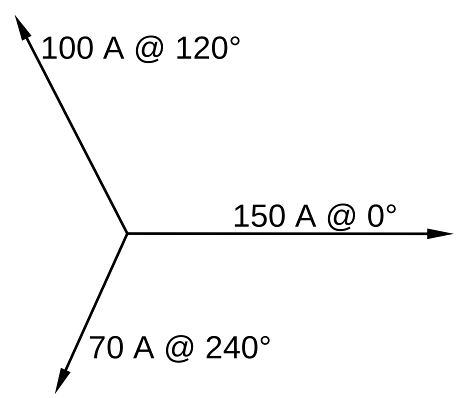
To find the direction of $\mathbf{C_R}$:

$$\theta = \tan^{-1} \frac{-2.7}{13} = -11.7°$$

The resultant pulling line current
is 13.3 A @ 348.3° (standard position).

Example 10

Find the resultant current in a three-phase AC circuit when the three line currents are 150 A at 0°, 100 A at 120°, and 70 A at 240°.



vector in polar form	horizontal component	vertical component
150 A @ 0°	150 · cos 0° = 150 A	150 · sin 0° = 0 A
100 A @ 120°	100 · cos 120° = −50 A	100 · sin 120° = 86.6 A
70 A @ 240°	70 · cos 240° = −35 A	70 · sin 240° = −60.6 A
total	65 A	26 A

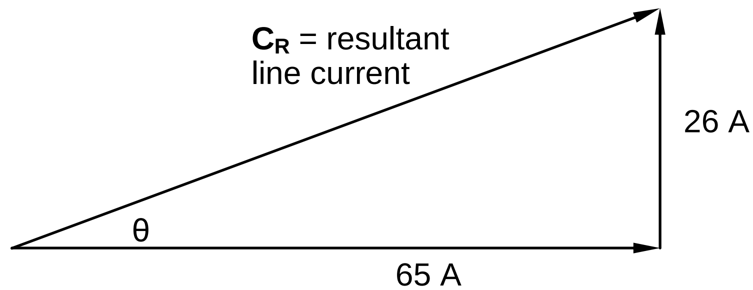
Combining the horizontal and vertical totals into one resultant current vector $\mathbf{C_R}$:

$$\mathbf{C_R} = \sqrt{(65)^2 + (26)^2} = 70 \text{ A}$$

To find the direction of $\mathbf{C_R}$:

$$\theta = \tan^{-1} \frac{26}{65} = 21.8°$$

The resultant pulling line current
is 70 A @ 21.8° (standard position).



Problem Set A

- Before writing the final unit 6 test on Vectors, make sure you can correctly solve all of the following problems (express all resultant vectors in polar form).
- If you have any difficulties solving these problems, complete Problem Set B.
- Answers are located at the end of this unit.

1. A plane flies due east for 95 km, makes a 90° turn, and then flies due south for 150 km. Find the plane's resultant displacement from its starting point (both distance and direction).

2. A boat sails due west at 8.5 km/hr, but it is pushed off course by a 7 km/hr current from the north. Find the resultant velocity of the sailboat (both rate of speed and direction).

3. Determine the resultant line current in a parallel AC circuit, when the two branch currents are 5.2 amps at 0° and 8.8 amps at 90°.

4. A ship travels 250 kilometres in a direction 31.9° north of west. Find the west and north components of its displacement.

5. A cable exerts a force of 1500 newtons (N) at an angle of 25° with the horizontal. Resolve this force into its horizontal component and its vertical component, which is the maximum weight that can be lifted by the cable.

6. Find the horizontal and vertical components of a voltage of 120 V at 240°.

7. A ship leaves port and sails 130 km in a direction 32° north of east, then turns and sails 75 km in a direction 65° north of west. Find the resultant displacement of the ship (both distance and direction).

8. Two tugboats are pulling a freighter. The largest tug exerts a pulling force of 14000 newtons due east, while the smaller tug exerts a force of 9500 newtons in a direction 35° south of east. Find the resultant pulling force of both tugs (both magnitude and direction).

9. Find the resultant current in a three-phase AC circuit when the three line currents are 4 A at 0°, 5 A at 90°, and 8 A at 225°.

10. Find the resultant current in a three-phase AC circuit when the three line currents are 120 A at 0°, 100 A at 150°, and 90 A at 300°.

Problem Set B

- Express all resultant vectors in polar form.
- If you have any difficulties solving these problems, complete Problem Set C.
- Answers are located at the end of this unit.

1. A ship sails due south for 230 km, then turns and sails due west for 645 km. Find the ship's displacement from its starting point (both distance and direction).

2. A boater wants to follow a course 24.8° south of west, when the current is flowing due south at 21.5 km/hr. Find (a) at what speed the boat should head due west, and (b) the resultant speed of the boat at 24.8° S of W.

3. Determine the resultant voltage in an AC circuit, when the two branch voltages are 110 volts at 0° and 60 volts at 90°.

4. A plane leaves an airport and flies 126 km in a direction 64.4° south of west. Find the west and south components of its displacement from the airport.

5. The wind is blowing at 74.5 km/hr in a direction 40° south of west. Find the west and south components of its velocity.

6. Find the horizontal and vertical components of a current of 80 A at 150°.

7. A plane is flying due south with an airspeed of 523 km/hr. The wind is from the southeast (45° south of east) at 94.5 km/hr. Find the plane's resultant velocity (both groundspeed and actual direction of travel).

8. A tractor and a truck are trying to pull a car out of a snowbank, with respective forces of 8160 N at 105° and 6200 N at 120°. Find the magnitude and direction of the resultant pulling force.

9. Find the resultant voltage in a three-phase AC circuit when the three line voltages are 75 V at 0°, 60 V at 30°, and 50 V at 300°.

10. Find the resultant current in a three-phase AC circuit when the three line currents are 55 A at 90°, 44 A at 210°, and 78 A at 315°.

Problem Set C

- Express all resultant vectors in polar form.
- Answers are located at the end of this unit.

1. A cyclist travels 14 km north, then she makes a left turn and cycles 23.4 km west. Find her displacement from the starting point (both distance and direction).

2. A boater wants to go straight across a river that has a current of 7.5 km/hr. If the boat can travel at 28 km/hr in still water, find (a) at what angle to the shore the boater should head and (b) the resultant speed of the boat.

3. Determine the resultant line current in a parallel AC circuit, when the two branch currents are 4.5 amps at 0° and 9.2 amps at 90°.

4. A plane is flying at 530 km/hr in a direction 38.5° north of west. Find the west and north components of its velocity.

5. A cable exerts a force of 456 N at an angle of 48.7° with the horizontal. Resolve this force into its horizontal and vertical components.

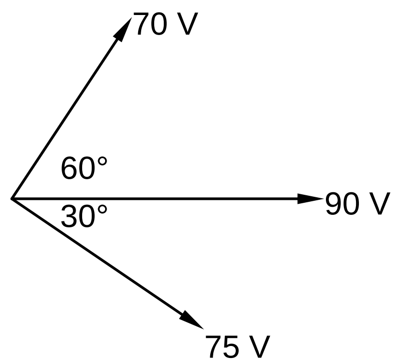
6. Find the horizontal and vertical components of a voltage of 110 V at 320°.

7. A boat sails at 5.6 km/hr in a direction 29° north of east, but it is pushed off course by a 4.7 km/hr wind blowing at 73° north of east. Find the resultant velocity of the sailboat (both rate of speed and direction).

8. A pilot is heading his plane due east with an airspeed of 632 km/hr. The wind is from the northeast at 102.5 km/hr. Find the plane’s resultant velocity (both groundspeed and actual direction of travel).

9. Find the resultant voltage in a three-phase AC circuit when the three line voltages are 120 V at 90°, 110 V at 180°, and 150 V at 315°.

10. Find the resultant voltage (both magnitude and angle) for the vectors shown below.



Answers

Problem Set A

1. 177.6 km @ 302.3° (@57.7° S of E)
2. 11.0 km/hr @ 219.5° (@39.5° S of W)
3. 10.2 A @ 59.4°
4. 212.2 km W; 132.1 km N
5. 1359 N horizontal; 634 N vertical
6. −60 V horizontal; −103.9 V vertical
7. 157.8 km @ 60.1°
8. 22453 N @ 346°
9. 1.8 A @ 202.4°
10. 83.2 A @ 340.4°

Problem Set B

1. 684.8 km @ 199.6°
2. (a) 46.5 km/hr (b) 51.2 km/hr
3. 125.3 V @ 28.6°
4. 54.4 km W; 113.6 km S
5. 47.9 km/hr S; 57.1 km/hr W
6. −69.3 A horizontal; 40 A vertical
7. 461.1 km/hr @ 261.7°
8. 14239 N @ 111.5°
9. 152.6 V @ 355°
10. 28.0 A @ 307.6°

Problem Set C

1. 27.3 km @ 149.1°
2. (a) 74.5° (b) 27.0 km/hr
3. 10.2 A @ 63.9°
4. 414.8 km/hr W; 329.9 km/hr N
5. 301.0 N horizontal; 342.6 N vertical
6. 84.3 V horizontal; −70.7 V vertical
7. 9.6 km/hr @ 48.8°
8. 564.2 km/hr @ 352.6°
9. 14.4 V @ 105.7°
10. 191.4 V @ 6.9°