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Academic and Career Foundations Department

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Trigonometry

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Adult Basic Education
Intermediate Level Mathematics

Module 14
Trigonometry

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Contents

Learning outcomes

Trigonometry is an application of ratio and proportion, geometry, algebra, and measurement. Though it is usually considered an advanced math topic, it is applied in various levels of trades and technology.

When you have completed this module, you should be able to:

- name the parts of a right triangle
- find the missing side of a right triangle using the Pythagorean Theorem
- find the measure of an unknown side or angle of a right triangle using sine, cosine or tangent ratios
- solve problems using right triangle trigonometry

Procedure for independent study

You will require a scientific calculator, ruler, and protractor.

1. Study each of the units in order, complete all of the exercises, and check your answers. If you need assistance, contact your instructor.

2. Review the definitions and terminology in the Glossary (p 42).

3. For more practice on application problems, complete Problem Sets A & B (p 38−40).

4. Complete the Practice Test (p 36) and check your answers.

5. Complete the MATH 052 unit 5 test.

Introduction

Trigonometry means “triangle measure”. It is a branch of mathematics which was originally developed to calculate unknown sides and angles in triangles. Trigonometry is very useful in applications such as surveying, navigation, and electronics.

Example 1

Suppose you wanted to know the height of a tree without having to climb the tree with a measuring tape. You can find the height by taking two measurements from the ground (Figure 1).

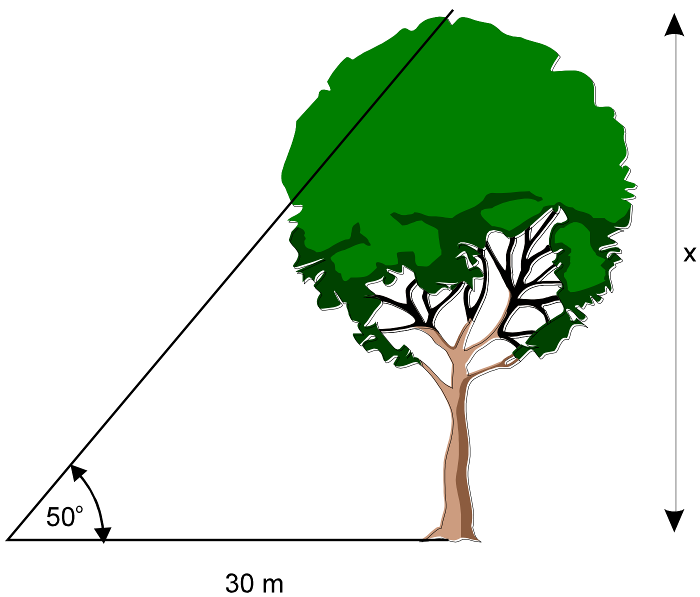


Figure 1

Once the distance from the base of the tree is known, as well as the angular measurement to the top of the tree, you can make a scale drawing of a similar triangle and measure the length of the corresponding side (Figure 2).

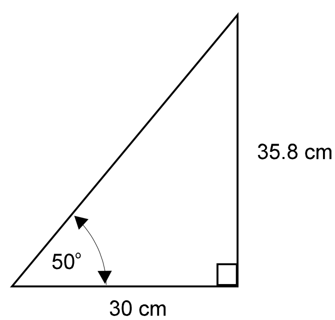


Figure 2

Using the proportion:

$$\frac{x}{30\text{ m}} = \frac{35.8\text{ cm}}{30\text{ cm}}$$

the tree is 35.8 m tall.

By using trigonometry, you can solve problems like this in an even simpler fashion.

5.1 The right triangle

The right triangle in Figure 3 is labelled using standard lettering.

The three angles of the triangle are $\angle A$, $\angle B$ and $\angle C$. The right angle is $\angle C = 90°$.

The sides of the triangle are a, b and c. The line segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ also denote the sides.

Side c is called the hypotenuse of the triangle while sides a and b are called the legs of the triangle. The hypotenuse is always the longest side of the triangle.

The Pythagorean Theorem states that for any right triangle,

$$c^2 = a^2 + b^2$$

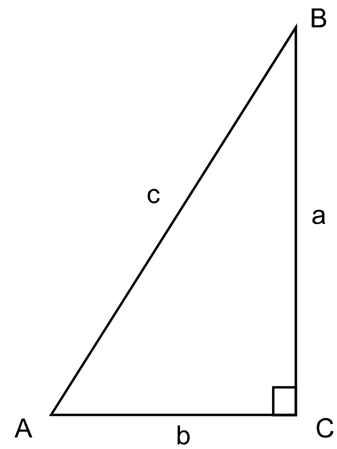


Figure 3

Angles of triangles are designated with capital letters. Sides are designated with lower case letters.

In a right triangle (Figure 4), sides are related to angles as follows:

- side a is opposite $\angle A$
- side b is opposite $\angle B$
- side c is opposite $\angle C$

In a right triangle, the hypotenuse is always opposite the right angle. Note that the names of angles A and B could be reversed and the same for sides a and b. Only side c and angle C are fixed.

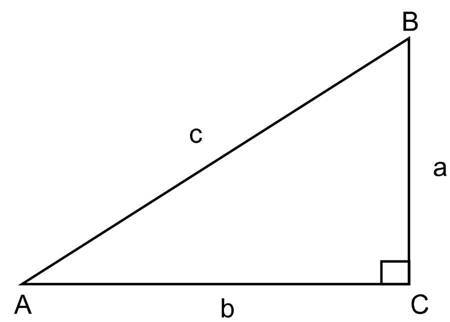


Figure 4

Notice that

- the side opposite an angle is not part of the angle
- side a is adjacent to $\angle B$
- side b is adjacent to $\angle A$

Sides that are adjacent to angles are actually part of the angle. The hypotenuse is already named and is not considered to be an adjacent side of an angle. Also, we do not consider the right angle to have adjacent sides.

Now complete Exercise 5.1 and check your answers. You may have to sketch triangles and take measurements with a ruler and protractor to answer some questions.

Exercise 5.1

1. All triangles have three sides and three angles.

   a. true b. false

2. The sum of the measures of the three angles of any triangle is equal to 180°.

   a. true b. false

3. The sum of the length of any two sides of a triangle is always longer than the third side.

   a. true b. false

4. The side opposite the smallest angle of a triangle has the shortest length. The side opposite the largest angle has the longest length.

   a. true b. false

5. The perimeter of a triangle is equal to the product of ½ the base times the height.

   a. true b. false

6. The area of a triangle is equal to the product of ½ the base times the height.

   a. true b. false

7. All right triangles have one right angle.

   a. true b. false

8. The side opposite the right angle is called the hypotenuse.

   a. true b. false

9. The sides that form the right angle are perpendicular to each other.

   a. true b. false

10. The Pythagorean Theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

    a. true b. false

Check your answers with the answer key, located at the end of this unit.

5.2 Angles and sides

The sum of the interior angles of any triangle is 180°. In any right triangle (Figure 5), one of the angles is 90°. The sum of the other two angles is also 90° or

$$\angle A + \angle B = 90°$$

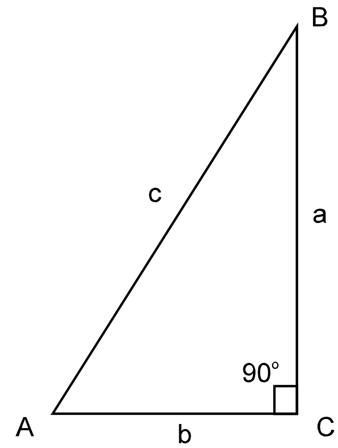


Figure 5

Example 1

Find $\angle B$ of the triangle in Figure 6.

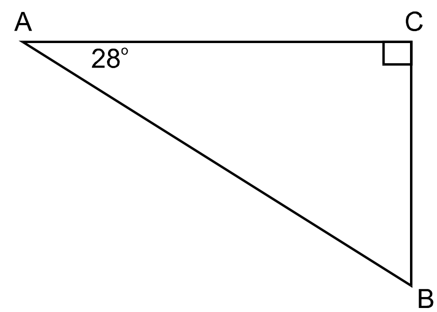


Figure 6

Solution

Using the formula:

$$\angle A + \angle B = 90°$$

$$28° + \angle B = 90°$$

$$\angle B = 90° - 28°$$

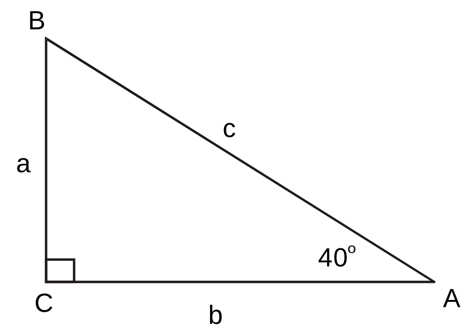
$$\angle B = 62°$$

It is very important that you always know precisely which angles and sides are involved in your calculations.

Now complete Exercise 5.2 and check your answers.

Exercise 5.2

1. In the triangle shown, indicate:



   a. which side is the hypotenuse

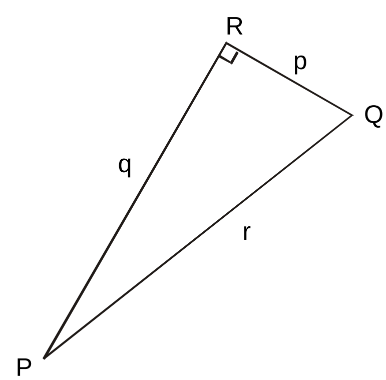
   b. which side is opposite ∠B

   c. which side is adjacent to ∠A

   d. which side is adjacent to ∠B

   e. the measure of ∠B

2. In ΔPQR, indicate:



   a. the hypotenuse

   b. the side opposite ∠Q

   c. the side adjacent to ∠P

   d. the side adjacent to ∠Q

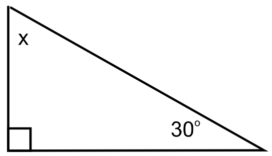
   e. the side opposite ∠P

   f. the side opposite ∠R

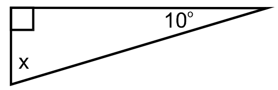
   g. if ∠P = 20° then ∠Q =

3. Determine ∠x in each of the following triangles.

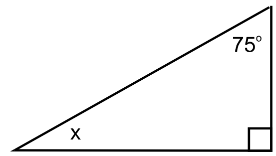
a.



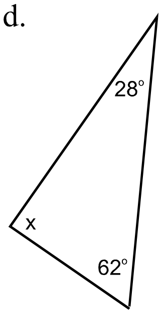
b.



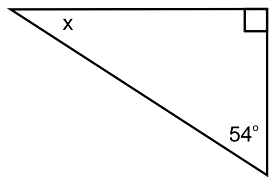
c.



d.



e.



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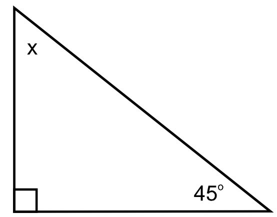
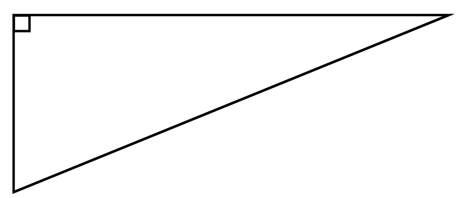


4. In the right triangle shown, identify the large (L), middle (M) and small (S) angles. Also identify the long (l), middle (m) and short (s) sides. Then answer the following questions:



a. Which angle is the long side opposite?

b. Which angle is the middle side opposite?

c. Which angle is the short side opposite?

d. Are the above statements true for any triangle?

Check your answers with the answer key at the end of this unit.

5.3 The Pythagorean Theorem

The Pythagorean Theorem is probably the most famous theorem in mathematics. It was known to the ancient Chinese, Egyptians and Greeks thousands of years ago.

Pythagoras' Theorem (Figure 7) states that the square of the hypotenuse of a right triangle equals the sum of the square of its legs or:

$$c^2 = a^2 + b^2$$

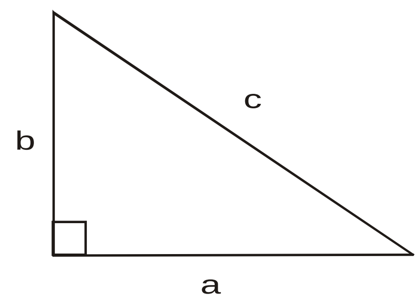


Figure 7

This theorem allows us to find the length of the third side of any right triangle when we know the length of two of the other sides.

Example 1

Find the hypotenuse of the triangle in Figure 8. Round your answer to one decimal place.

Solution

Using the formula

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 4.5^2$$

$$c^2 = 36 + 20.25$$

$$c^2 = 56.25$$

$$c = \sqrt{56.25}$$

$$c = 7.5$$

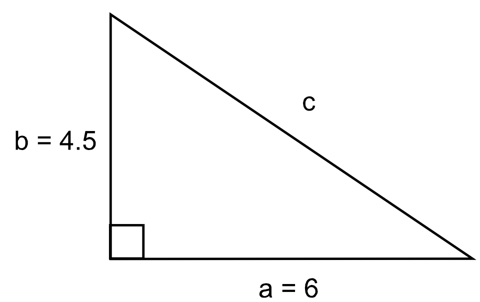


Figure 8

Example 2

Find the missing side of the right triangle in Figure 9. Round your answer to one decimal place.

Solution

The missing side of the right triangle is a leg. It can be named a or b. For this example, you can name it b.

$c^2 = a^2 + b^2$

$13.2^2 = 11.5^2 + b^2$

$174.24 = 132.25 + b^2$

$174.24 - 132.25 = b^2$

$41.99 = b^2$

$b = \sqrt{41.99}$

$b = 6.4799\ldots$

$b \approx 6.5$

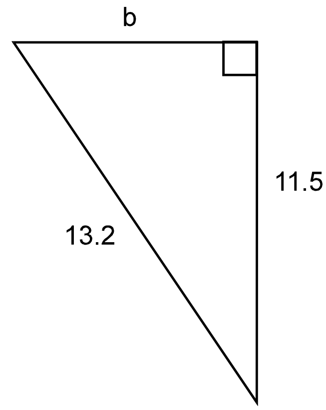


Figure 9

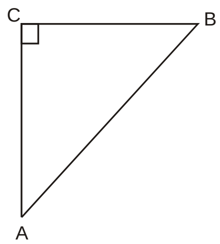
Note that many of the answers that we will calculate will be irrational numbers (decimal numbers that do not terminate and do not repeat in a predictable way). These numbers are "approximate" numbers, which can be rounded to a specified number of decimal places.

	Now complete Exercise 5.3 and check your answers.

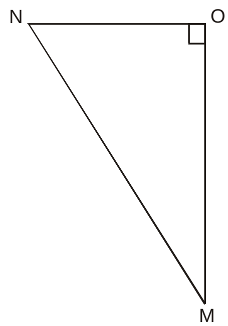
Exercise 5.3

1. Name the right angle in each of the triangles shown below.

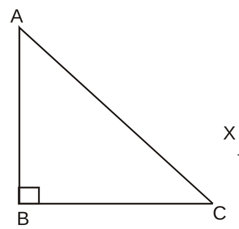
a.



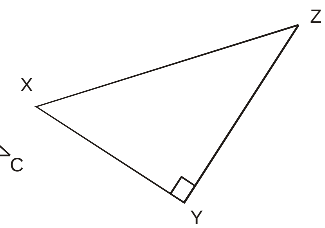
b.



c.

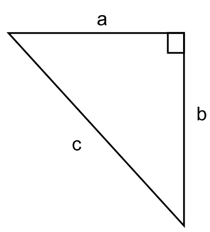


d.



2. Name the hypotenuse in each triangle shown below.

a.



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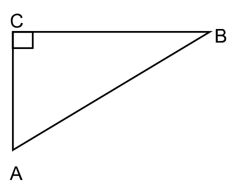

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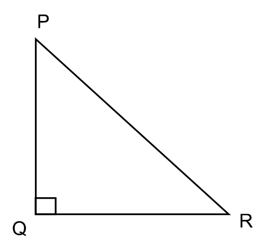

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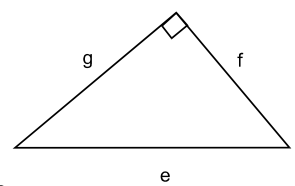
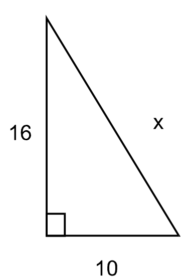


3. Find the missing side, x, in each triangle shown below. Round your answers to one decimal place.

a.



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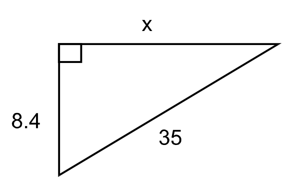
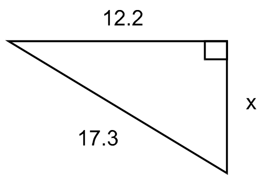


c.



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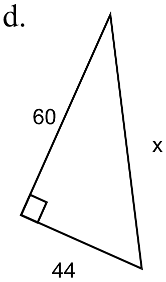
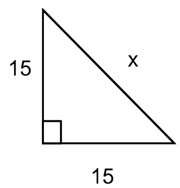


e.



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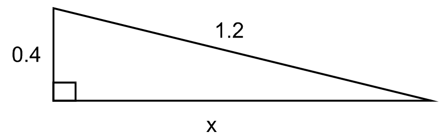
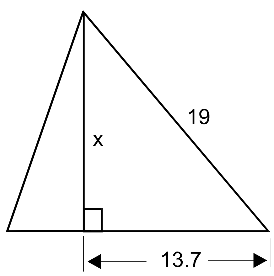
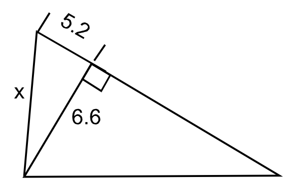


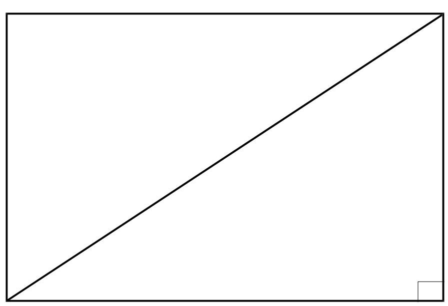
g.



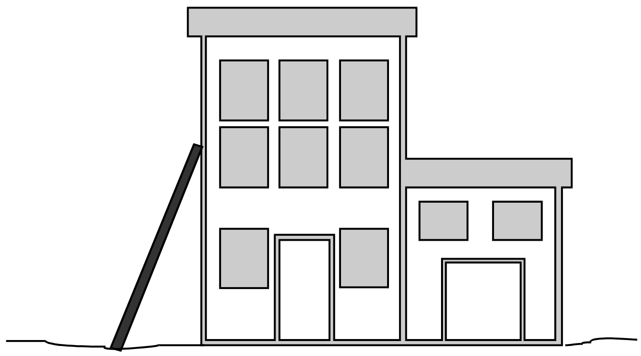
h.



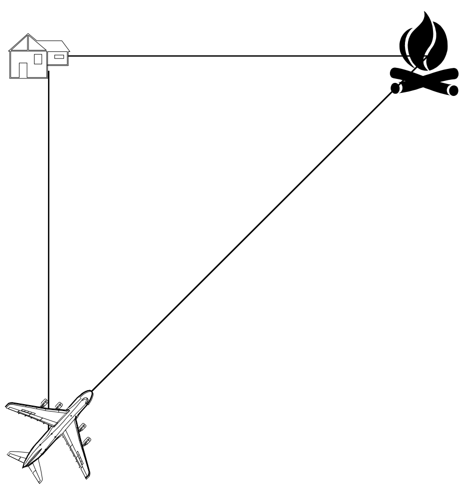
4. The length of a rectangle is 52 cm and the width is 40 cm. Find the length of the diagonal. Round your answer to one decimal place.



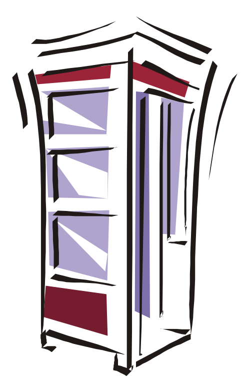
5. A 4 m ladder leans against a building. If the bottom of the ladder is 1.3 m from the bottom of the building, how far up the wall will the ladder reach? Round your answer to one decimal place.



6. A warden observes a fire 4.8 km directly east of his cabin. The nearest water bomber is stationed 10.5 km directly south of the cabin. How far is the water bomber from the fire? Round your answer to one decimal place.



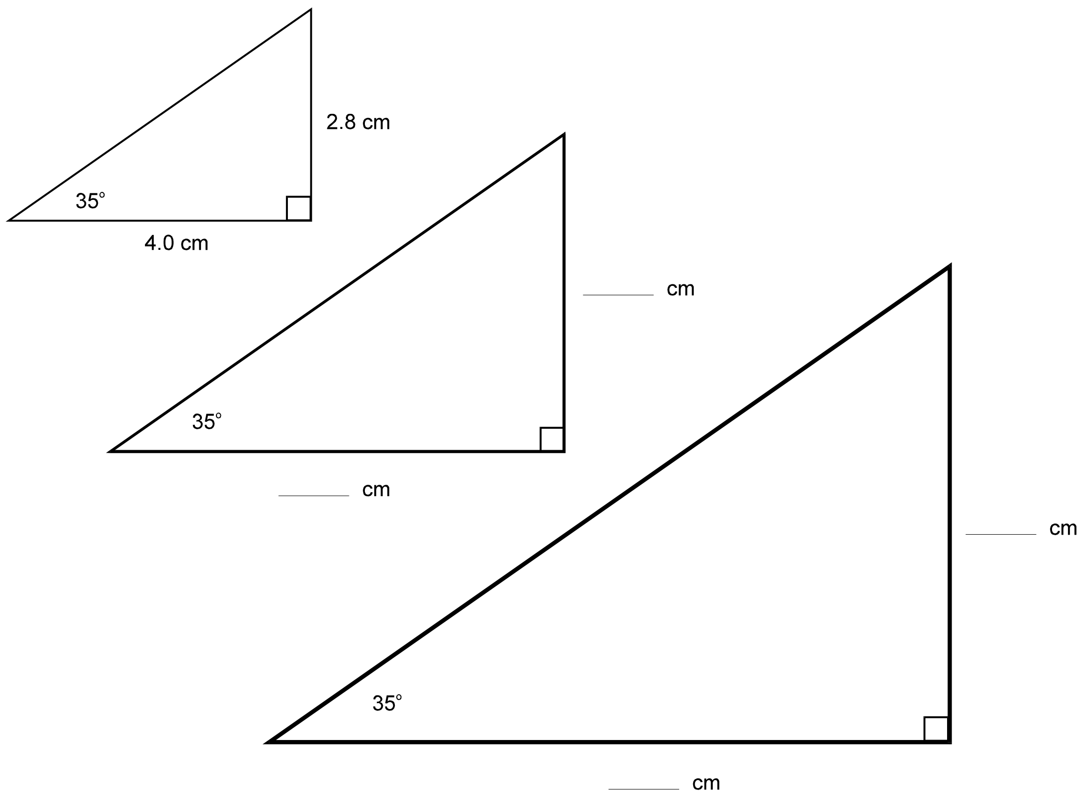
7. A basketball player is 2.3 m tall. He has to make a phone call from a pay phone booth which is 1 m wide, 1 m long and 2 m high. Can he fit himself into the phone booth without sitting down or bending? (Hint: What is the distance between opposite corners of the phone booth?)



Answer key at the end of this unit.

5.4 The tangent ratio

The three right triangles in Figure 10 are called similar triangles. They are similar triangles because their corresponding angles are equal. Notice that their sides are not equal. With a metric ruler, measure the opposite and adjacent sides (to the 35° angle) in each triangle and record your results in the table. Round your answers to 0.1 cm.



TRIANGLE	OPPOSITE SIDE	ADJACENT SIDE	RATIO OF $\frac{\text{OPPOSITE}}{\text{ADJACENT}}$	DECIMAL EQUIVALENT
1	2.8 CM	4.0 CM	$\frac{2.8}{4.0}$	0.7
2				
3				

Figure 10

The table should show that the ratio of opposite side to adjacent side is the same for all three triangles with a 35° angle, regardless of the size of the triangle. This is the core idea of trigonometry.

For any acute angle of a right triangle, the ratio of the opposite side to the adjacent side (Figure 11) is called the tangent ratio, abbreviated "tan".

For any right triangle:

$$\tan \angle A \text{ or } \tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

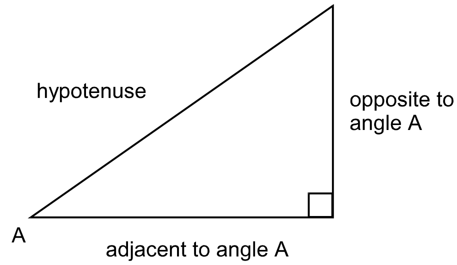


Figure 11

From your investigation of the three triangles in Figure 10, you can now write:

$$\tan 35° = \frac{28}{40} = 0.7$$

You can use a scientific calculator to determine the value of tan 35° by pressing
35 [TAN] or [TAN] 35 depending on your particular make of calculator.

Make sure your calculator is in degree mode (DEG) first.

You should get approximately 0.700208 (to nearest millionth).

Example 1

Use a metric ruler to determine tan 60° in Figure 12. Also determine tan 60° using your calculator.

Solution

Measure the opposite and adjacent sides of the triangle, then:

$$\tan 60° = \frac{4.6}{2.7} = 1.7$$

The calculator will show 1.732051 for tan 60°.

Again, note that tangent values to the nearest millionth in your calculator are much more accurate than the calculated values based on measurements to the nearest tenth.

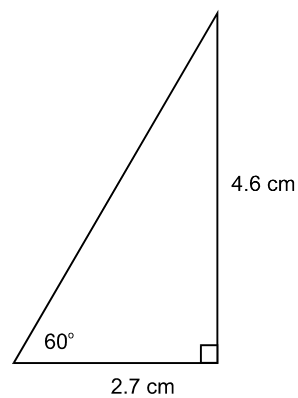
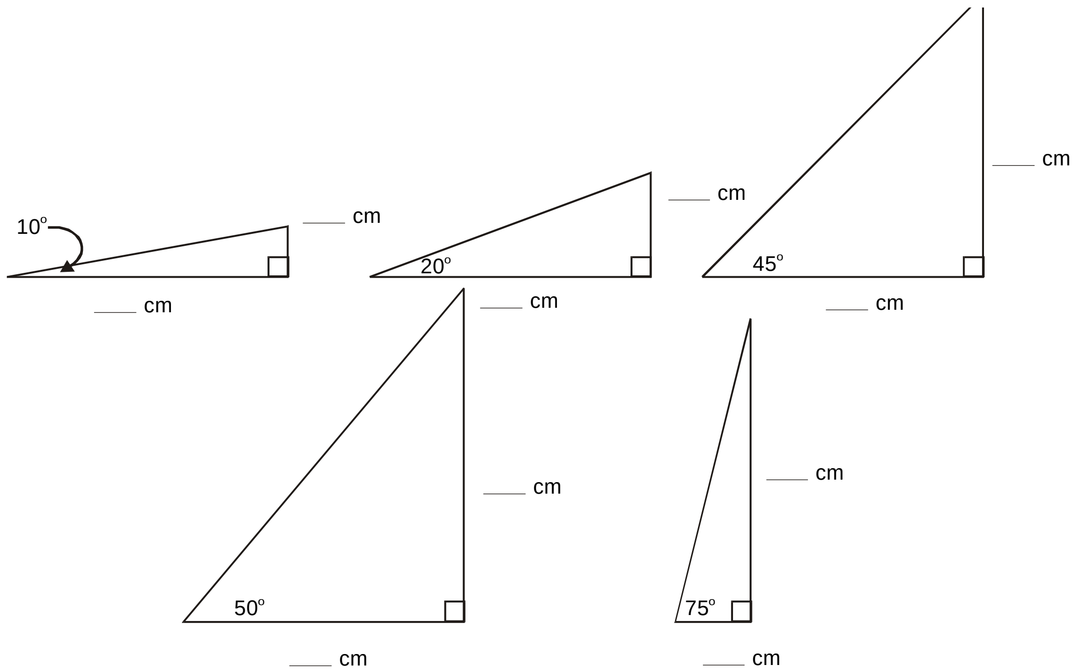


Figure 12

Now complete Exercise 5.4 and check your answers.

Exercise 5.4

1. Use a ruler, then a calculator to find the tangents of the indicated angles. Record your results in the table provided.



Angle	length of opposite side / length of adjacent side	decimal equivalent to six decimal places	calculator value of tan A to six decimal places
10°			
20°			
45°			
50°			
75°			

2. Use a calculator to find the following. Round your answers to four decimal places.

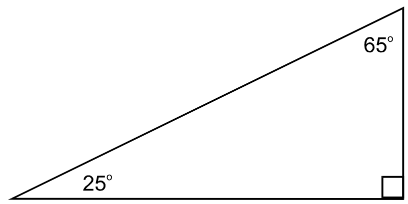
a. tan 0° b. tan 1° c. tan 5°

d. tan 15° e. tan 22° f. tan 31.4°

g. tan 45° h. tan 50.92° i. tan 64°

j. tan 0.05° k. tan 86° l. tan 89°

3. Measure the legs of the triangle to find the following. Round your answers to one decimal place.

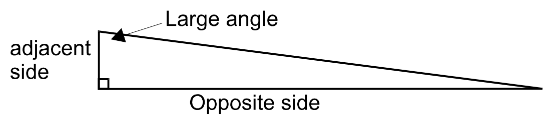


a. tan 25°

b. tan 65°

4. Use a calculator to find the following. Round your answers to three decimal places.

a. tan 88°



b. tan 89°

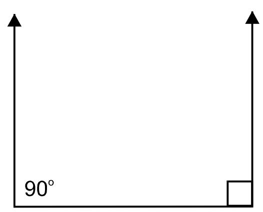
c. tan 89.9°

d. tan 89.99°

e. tan 90°

These values are very large. When a right triangle has a very large acute angle, the opposite side is much larger than the adjacent side.

5. Find tan 90°. Explain why tan 90° is undefined.



Answer key at the end of this unit.

5.5 Using the tangent ratio

If one acute angle and one leg of a right triangle are known, the tangent ratio can be used to find the other leg of the triangle.

Example 1

Find side b in the triangle in Figure 13. Round your answer to one decimal place.

Solution

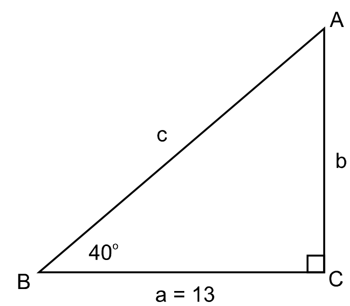


Figure 13

Side b is opposite 40° and the known side is adjacent to 40°, so we use the tangent ratio.

$$\tan 40° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{13}$$

Rewrite this as a proportion and solve by cross-multiplying.

$$\frac{\tan 40°}{1} = \frac{b}{13}$$

$1 \cdot b = 13 \cdot \tan 40°$ or $1 \cdot b = (\tan 40°) \cdot 13$

$b \approx 10.908 \approx 10.9$

To find $13 \cdot \tan 40°$, depending on the type of calculator,

press $13 \times \boxed{\text{TAN}}\ 40 =$ or $13 \times 40\ \boxed{\text{TAN}} =$

Example 2

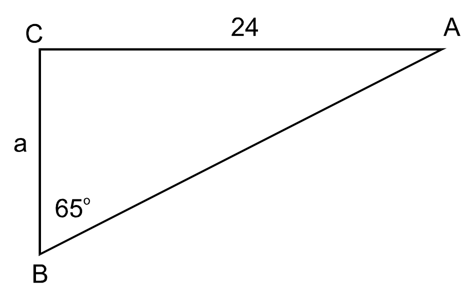


Figure 14

Find side a of the triangle in Figure 14. Round your answer to one decimal place.

Solution

Side a is adjacent to 65° and the known side is opposite 65°, so we use the tangent ratio.

$$\tan 65° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{24}{a}$$

Rewrite and solve as a proportion:

$$\frac{\tan 65^\circ}{1} = \frac{24}{a}$$

$$(\tan 65^\circ)\cdot a = 1\cdot 24 \qquad \text{cross - multiply}$$

$$a = \frac{24}{\tan 65^\circ} \approx 11.191 \qquad \text{divide by } \tan 65^\circ$$

$$a \approx 11.2$$

To find $\frac{24}{\tan 65^\circ}$, depending on the type of calculator,

press 24 ÷ $\boxed{\text{TAN}}$ 65 = or 24 ÷ 65 $\boxed{\text{TAN}}$ =

You can also use the tangent ratio to find an acute angle of the right triangle when both legs are known.

Example 3

Find angle A of the right triangle in Figure 15. Round your answer to the nearest degree.

Solution

The known sides are opposite and adjacent to angle A, so we use the tangent ratio.

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{16}{13} \approx 1.230769$$

To find angle A, with the calculator displaying its tangent of 1.230769, press $\boxed{\text{INV}}$ $\boxed{\text{TAN}}$ or $\boxed{\text{2ndF}}$ $\boxed{\text{TAN}}$ or $\boxed{\text{SHIFT}}$ $\boxed{\text{TAN}}$ or $\boxed{\text{TAN}^{-1}}$, to display the angle 50.906° or 50.9°.

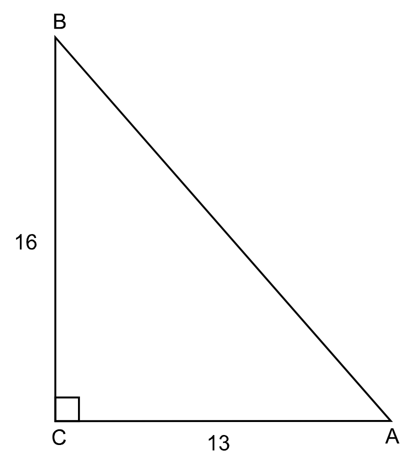


Figure 15

When the tangent of an angle is known, the angle is called the “inverse tangent” or “arc tangent”.

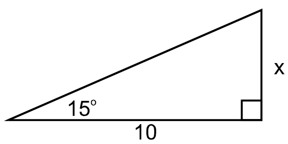
Now complete Exercise 5.5 and check your answers.

Exercise 5.5

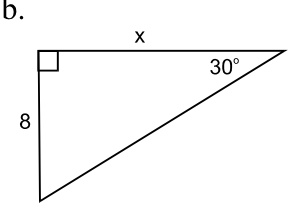
1. Find side x in each of the following triangles. Round your answer to one decimal place.

Remember: $\tan A = \dfrac{\text{opposite side}}{\text{adjacent side}}$.

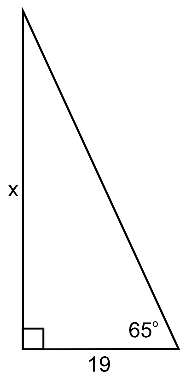
a.



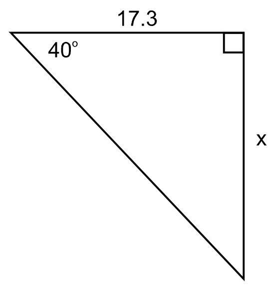
b.



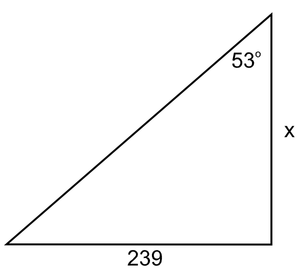
c.



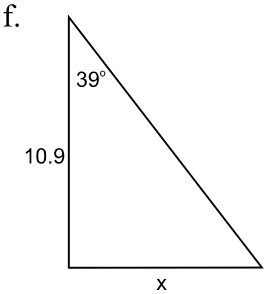
d.



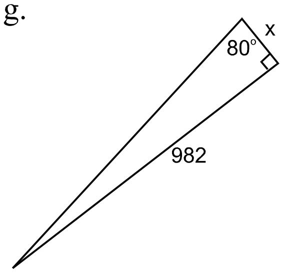
e.



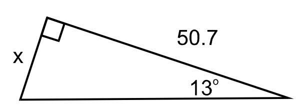
f.



g.



h.



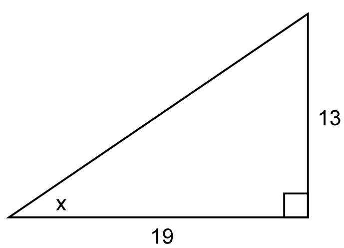
2. a. In ΔABC, $\angle C = 90°$, $\angle B = 55°$ and $\overline{AC} = 18$.
Sketch and label ΔABC, then find $\overline{BC}$.

b. In ΔABC, $\angle C = 90°$, $\angle A = 12°$ and $\overline{AC} = 43$.
Sketch and label ΔABC, then find $\overline{BC}$.

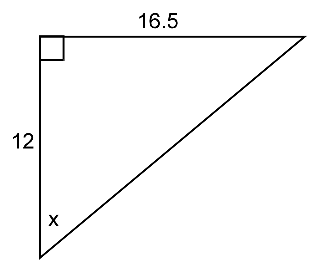
3. Find angle x in each of the following triangles. Round answers to one decimal place.

Remember: $\tan x = \dfrac{\text{opposite side}}{\text{adjacent side}}$.

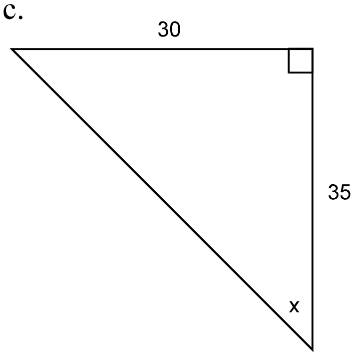
a.



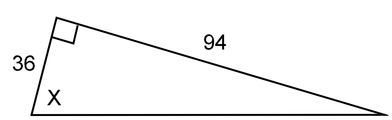
b.



c.



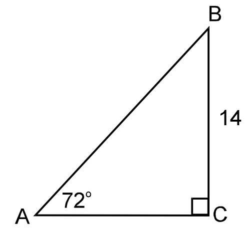
d.



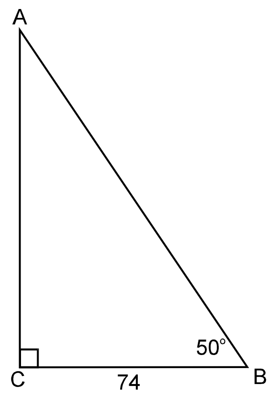
4. In ΔABC, $\angle C = 90°$, $\overline{AC} = 5.3$ and $\overline{BC} = 7.5$.
Sketch and label ΔABC, then find $\angle A$ and $\angle B$.

5. Find the missing angles and sides in ΔABC.

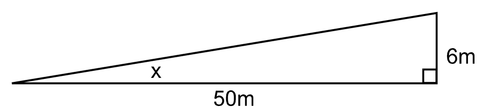
Hints: Use $\angle A + \angle B = 90°$ to find $\angle B$.
Use the tangent ratio to find $\overline{AC}$.
Use the Pythagorean Theorem to find $\overline{AB}$.



6. Solve ΔABC. (find $\angle A$, $\overline{AC}$ and $\overline{AB}$).

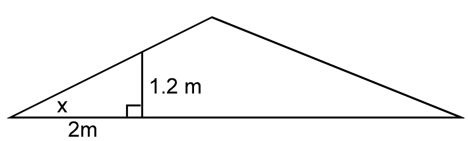


7. A road rises 6 m vertically for every 50 m of horizontal distance. At what angle is the road inclined?



8. Joan stands 20 m from the base of a large tree. Her line of sight to the top of the tree makes an angle of 54° with the horizontal. Joan is 1.6 m tall. How tall is the tree?

9. A roof rises 1.2 m for every 2 m run. Find the pitch of the roof in degrees (angle x).



Answer key at the end of this unit.

5.6 The sine and cosine ratios

The tangent ratio can be used to find missing sides of right triangles. If the hypotenuse of a right triangle is unknown, there are two other trigonometric ratios that are useful:

- the sine ratio, abbreviated sin, pronounced "sign"
- the cosine ratio, abbreviated cos, pronounced "kose"

For any right triangle (Figure 16):

$$\sin \angle A \text{ or } \sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \angle A \text{ or } \cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

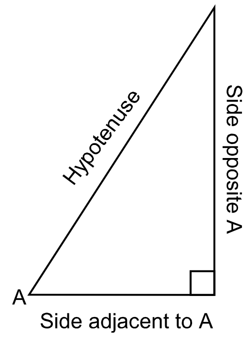


Figure 16

Example 1

Find side $\overline{AB}$ in ΔABC (Figure 17).

Solution

An angle and its opposite side are known. To find the hypotenuse, use the sine ratio.

$$\sin 41^\circ = \frac{8}{\overline{AB}}$$

Rewrite and solve as a proportion:

$$\frac{\sin 41^\circ}{1} = \frac{8}{\overline{AB}}$$

$$(\sin 41^\circ)\,\overline{AB} = 8$$

$$\overline{AB} = \frac{8}{\sin 41^\circ} \approx 12.194 \approx 12.2$$

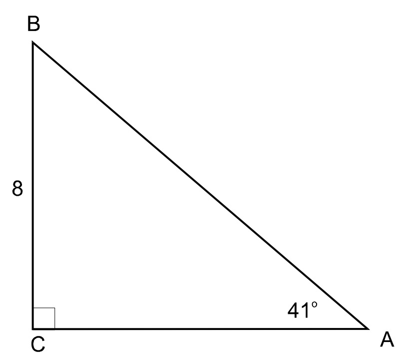


Figure 17

To find $\frac{8}{\sin 41^\circ}$ on a calculator,

press 8 ÷ $\boxed{\text{SIN}}$ 41 = or 8 ÷ 41 $\boxed{\text{SIN}}$ =

Example 2

Find side x of the triangle in Figure 18.

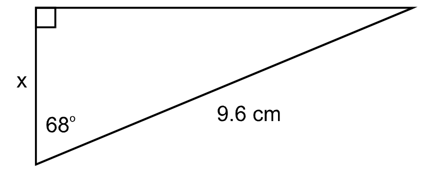


Figure 18

Solution

Side x is adjacent to the 68° angle and the hypotenuse is 9.6 cm. The cosine ratio can be used to find side x.

$$\cos 68° = \frac{x}{9.6}$$

$$\frac{\cos 68°}{1} = \frac{x}{9.6}$$

$$x = 9.6 \cos 68° = 3.596 \approx 3.6 \text{ cm}$$

To find 9.6 cos 68° on a calculator, press 9.6 × $\boxed{\text{COS}}$ 68 = or 9.6 × 68 $\boxed{\text{COS}}$ =

Example 3

Find ∠A of ΔABC when $\overline{AB} = 44$, $\overline{AC} = 29$ and $\angle C = 90°$.

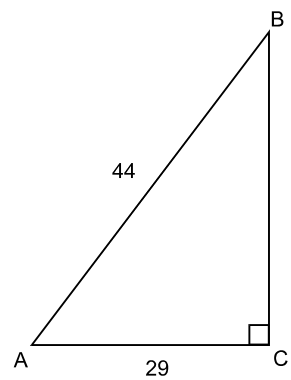


Figure 19

Solution

Sketch ΔABC and label the given parts (Figure 19). The hypotenuse and adjacent side to ∠A are given, so use the cosine ratio to find ∠A.

$$\cos A = \frac{29}{44} = 0.659$$

$$\angle A = 48.8°$$

To find ∠A, with the calculator displaying its cosine of 0.659, press $\boxed{\text{INV}}$ $\boxed{\text{COS}}$ or $\boxed{\text{2ndF}}$ $\boxed{\text{COS}}$ or $\boxed{\text{COS}^{-1}}$, to display the angle 48.8°.

When the cosine of an angle is known, the angle is called the "inverse cosine" or "arc cosine".

A common mnemonic (memory assisting device) to remember the sine, cosine, and tangent ratios is the word SOHCAHTOA (pronounced "sew caw toa"), which is shorthand for the ratios

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Now complete Exercise 5.6 and check your answers.

Exercise 5.6

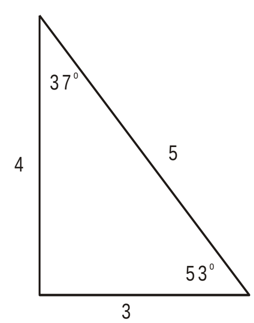
1. Use a calculator to find the following pairs of sine and cosine values.
Round answers to four decimal places.

	sin A	cos B
a)	sin 1°	cos 89°
b)	sin 5°	cos 85°
c)	sin 10°	cos 80°
d)	sin 22.5°	cos 67.5°
e)	sin 30°	cos 60°
f)	sin 45°	cos 45°
g)	sin 60°	cos 30°
h)	sin 67.5°	cos 22.5°
i)	sin 80°	cos 10°
j)	sin 85°	cos 5°
k)	sin 89°	cos 1°

Study both columns of answers. Notice that for each pair of values, $\angle A + \angle B = 90°$ ("complementary" angles), and sin A = cos B.

For any pair of complementary angles, the sine of one angle equals the cosine of the other, or in other words, $\sin A = \cos(90 - A)$.

2. From the triangle shown, find the following to one decimal place.



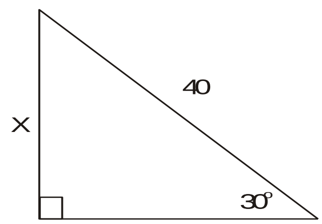
a. sin 37°
b. cos 53°
c. sin 53°
d. cos 37°
e. suppose that sin 20° = 0.342. What is cos 70°?

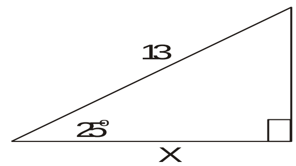
3. Find side x in each triangle. Round your answers to one decimal place. For each of the following triangles, ask yourself the following three questions:

1. Name side x. Is it the hypotenuse (H), the opposite side (O), or the adjacent side (A)?
2. Name the given side. Is it the hypotenuse, the opposite side, or the adjacent side?
3. Using SOHCAHTOA, which ratio should be used? sin, cos, or tan?

a. Name side x. Is it the hypotenuse, opposite, or adjacent to 30°? ___________
Name the given side 40. Is it the H, O or A side? ___________
Which ratio? sin, cos or tan?

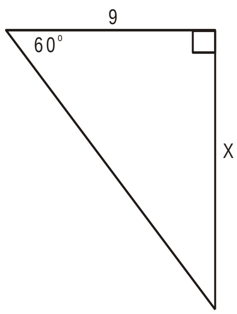
b. Name side x. H, O or A? ______
Name side 13. H, O or A? ______
Use sin, cos or tan? ______



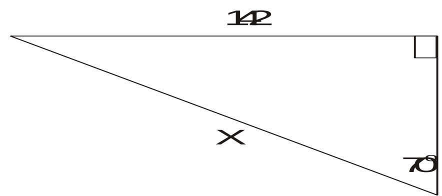


Ask and answer the above three questions for each triangle below.

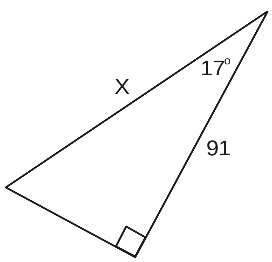
c.



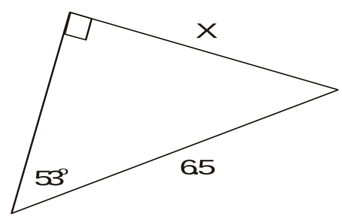
d.



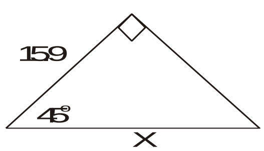
e.



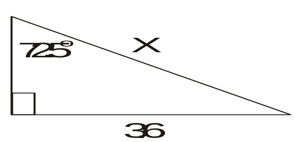
f.



g.



h.

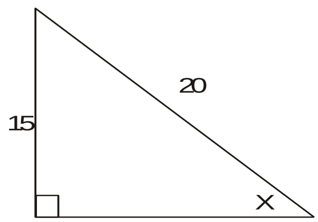


4. In ΔABC $\angle C = 90°$, $\angle B = 24°$ and $\overline{AC} = 75$ cm. Find $\overline{AB}$.

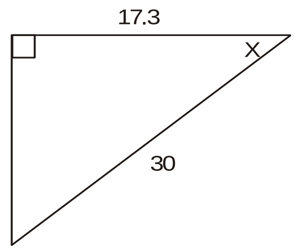
5. In ΔABC $\angle C = 90°$, $\angle A = 69°$ and $\overline{AB} = 24.7$ cm. Find $\overline{BC}$.

6. Find ∠x in each of the following. Round your answers to the nearest degree. (use SOHCAHTOA)

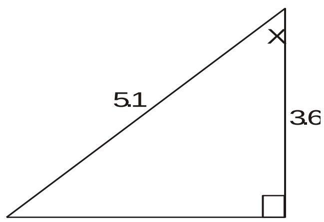
a.



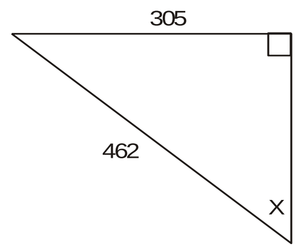
b.



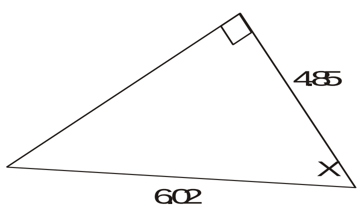
c.



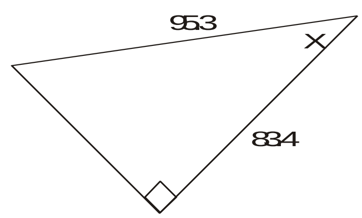
d.



e.

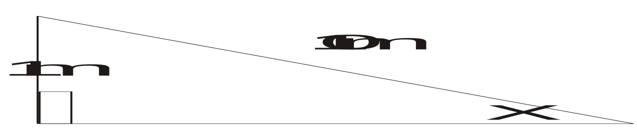


f.

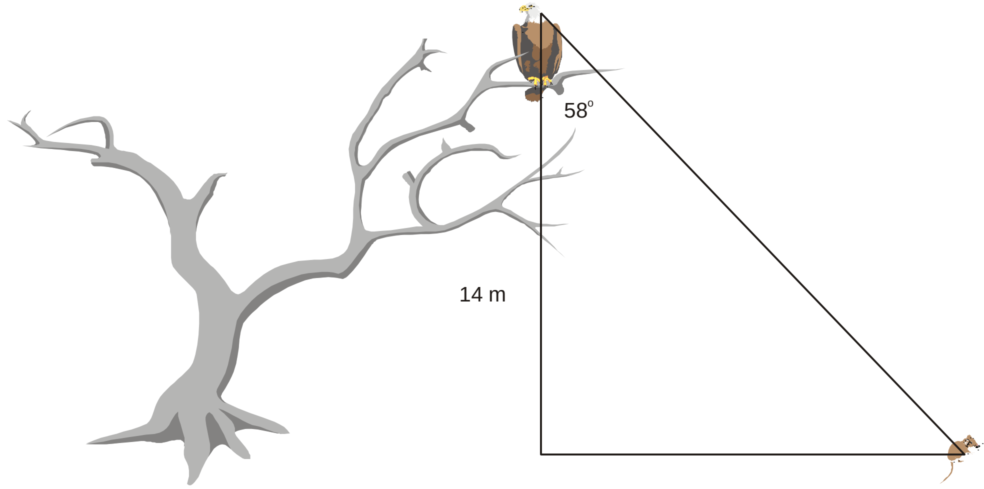


7. In ΔABC ∠C = 90°, $\overline{AC}$ = 14 and $\overline{AB}$ = 19. Find ∠A and ∠B to the nearest tenth of a degree.

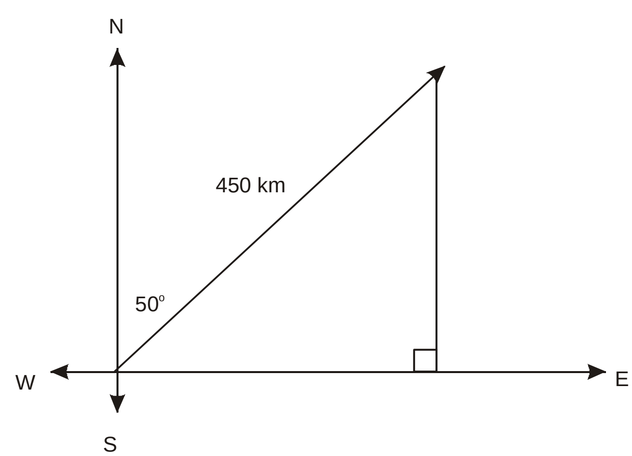
8. A 10 m long ramp rises to a point 1 m above the ground. Find the angle of inclination of the ramp.



9. An eagle is sitting on a tree 14 m above the ground. The eagle sights a mouse on the ground at an angle of 58° with the tree. How far is the eagle from the mouse?



10. An airplane travels at 450 km/hr in a N50°E direction. How far east has the plane travelled after 1 hour?



Answer key at the end of this unit.

5.7 Solving triangles

To “solve a triangle” means to determine all the unknown sides and angles. Before this is possible, you have to know three parts of the triangle. One part must be a side..

Example 1

Solve ΔABC where $\angle C = 90°$, $\angle A = 53°$ and $c = 18$.

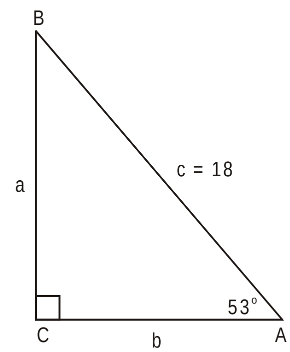


Figure 20

Solution

Sketch ΔABC and label the known parts (Figure 20), then find $\angle B$.

$$\angle A + \angle B = 90°$$
$$53° + \angle B = 90°$$
$$\angle B = 90° - 53°$$
$$\angle B = 37°$$

Find side a. Side a is opposite 53° and c = 18 is the hypotenuse. Using the sine ratio,

$$\sin 53° = \frac{a}{18}$$
$$18 \sin 53° = a$$
$$a \approx 14.4$$

There are several ways of determining side b. The safest method, which involves only given information, is to use the cosine ratio, since side b is adjacent to 53°.

$$\cos 53° = \frac{b}{18}$$
$$18 \cos 53° = b$$
$$b \approx 10.8$$

Both the Pythagorean Theorem and the tangent ratio require calculated answers to find side b. Even though they are valid methods, they are not the most reliable. You could use a formula such as:

$\cos 37° = \frac{a}{18}$ to find side a.

This requires that you involve the calculated value of 37°. If 37° is a wrong answer, then side a will also be a wrong answer. It is always more reliable to involve given information in a calculation whenever possible. Also note that if you round a value that you use in a subsequent calculation, the next value that you find will also be an approximation.

Example 2

Solve the triangle in Figure 21.

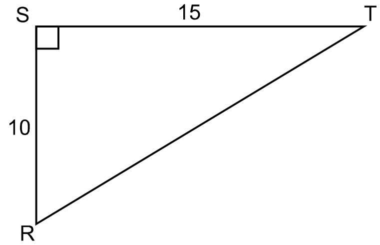


Figure 21

Solution

Find the hypotenuse, $\overline{RT}$ as follows:

$$\overline{RT} = 10^2 + 15^2$$
$$\overline{RT}^2 = 100 + 225 = 325$$
$$\overline{RT} = \sqrt{325} \approx 18.0$$

Find $\angle$R as follows; side 15 is opposite $\angle$R, side 10 is adjacent to $\angle$R, so use the tangent ratio,

$$\tan R = \frac{15}{10} = 1.5$$
$$\angle R \approx 56.3°$$

You can use the tangent ratio to find $\angle$T:

$$\tan T = \frac{10}{15}$$
$$\angle T \approx 33.7°$$

Although it is not as reliable, it is also just as easy to use:

$\angle T + \angle R = 90°$

$\angle T + 56.3° = 90°$

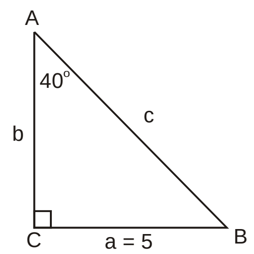
$\angle T = 90° - 56.3° = 33.7°$

	Now complete Exercise 5.7 and check your answers.

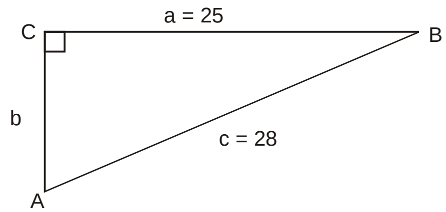
Exercise 5.7

1. Solve the following triangles. Find the angles to the nearest degree and the sides to one decimal place.

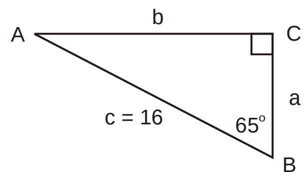
a.



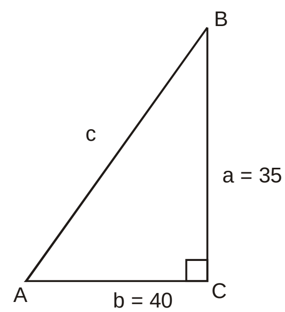
b.



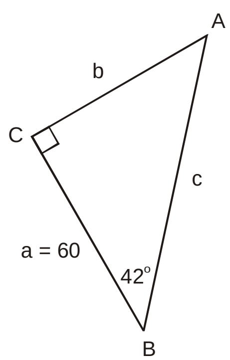
c.



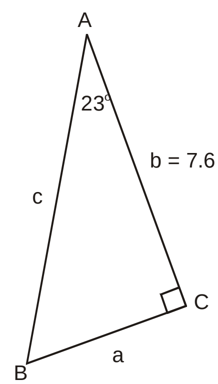
d.



e.



f.



2. Solve ΔABC where $\angle C = 90°$, $\angle A = 16°$ and $b = 92$.

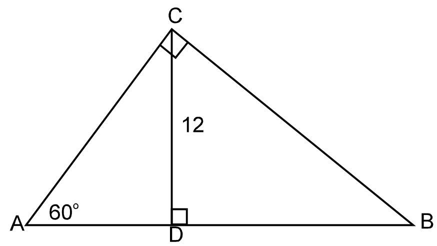
3. Solve ΔABC where $\angle C = 90°$, $a = 12$ and $c = 13$.

4. Solve ΔABC where $\angle C = 90°$, $a = 1$ and $b = 1$.

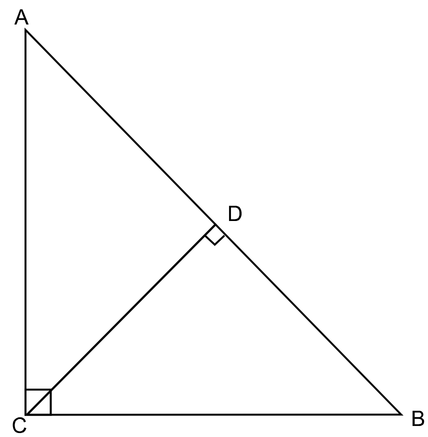
5. Solve ΔABC where $\angle C = 90°$, $\angle A = 40°$ and $\angle B = 50°$.

6. In ΔABC, note that $\angle ACB = 90°$, $\angle A = 60°$ and altitude $\overline{CD} = 12$. Find the following:

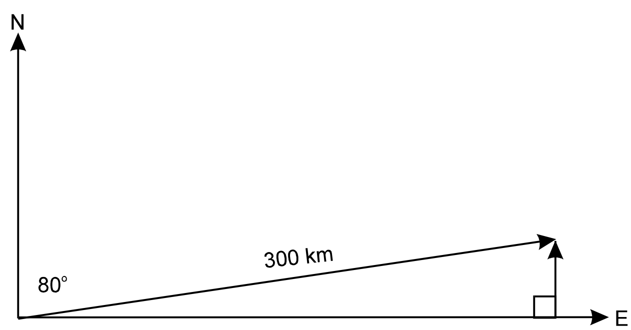
a. $\angle B$
b. $\overline{AC}$
c. $\overline{BC}$
d. $\overline{AB}$



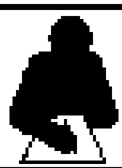
7. In ΔABC, $\angle ACB = 90°$, $\angle A = 65°$ and $\overline{AB} = 20$. Find altitude $\overline{CD}$.



8. An airplane flies 300 km at a heading of 80°. How far north and how far east has it travelled?



Answer key at the end of this unit.

	Now complete the Practice Test and check your answers.

Practice Test

1. Use a calculator to find the following. Round your answers to four decimal places.

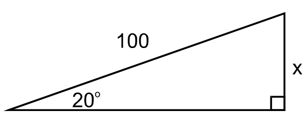
   a. sin 36°
   b. cos 9.2°
   c. tan 87°
   d. sin 0.49°

2. Use a calculator to find ∠A in each of the following. Round your answers to the nearest degree.

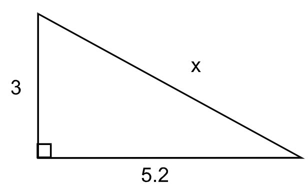
   a. cos A = 0.9889
   b. tan A = 1.5901
   c. sin A = $\frac{6}{11}$
   d. tan A = 3

3. Find side x or angle x in each of the following. Round your answers to one decimal place. Remember SOHCAHTOA.

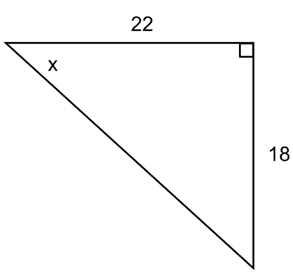
a.



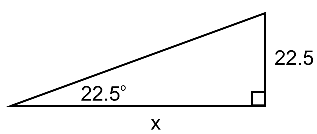
b.



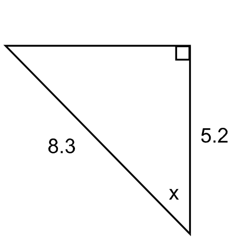
c.



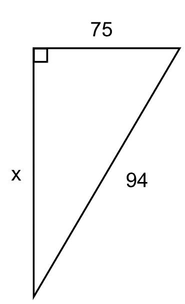
d.



e.



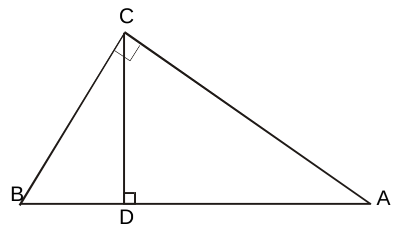
f.



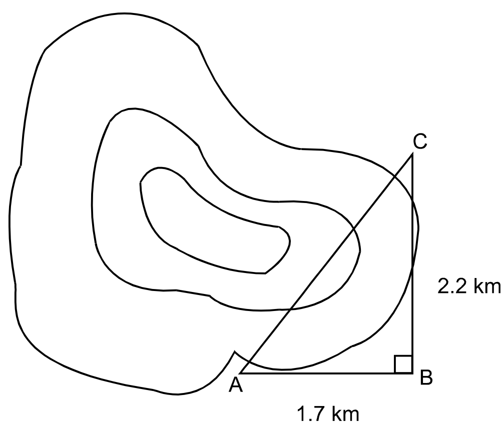
4. Solve ΔABC where $\angle C = 90°$, $\angle B = 40°$ and $a = 88$.

5. Solve ΔABC where $\angle C = 90°$, $a = 24$ and $c = 30$.

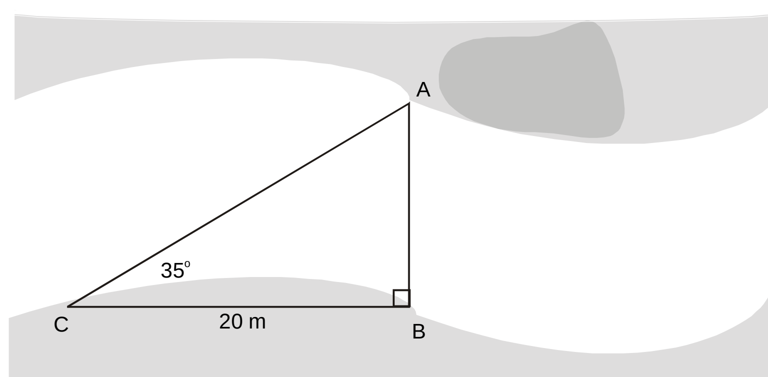
6. Find the altitude $\overline{CD}$ of ΔABC if $\angle A = 25°$ and $\overline{AB} = 1.2$. Note that ΔABC is a right triangle.



7. Engineers measure the right-angled distances beside a mountain as shown. They propose to build a tunnel from A to C through the mountain. What is the length of the tunnel?



8. Grace wants to know how far it is to the other side of the river. She makes the measurements as shown. What is the distance from A to B across the river?



Answer key at the end of this unit.

Problem Set A

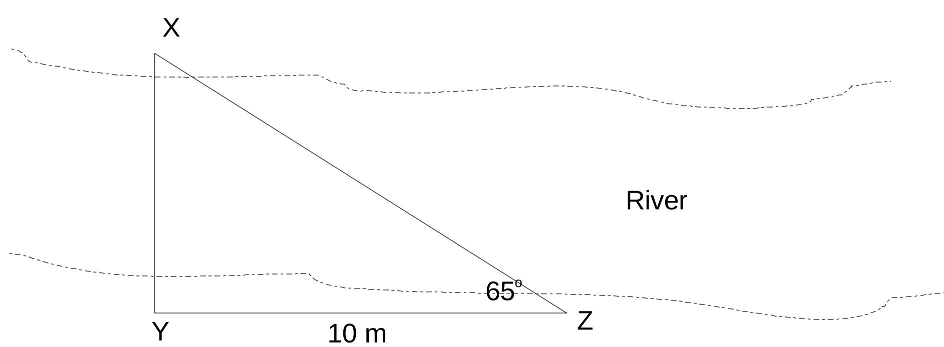
Read each question carefully, then draw a diagram. When you have finished the diagram, read the question again to make sure you have interpreted it properly, then work out the solution.

1. Pat wants to know the height of a tree in her yard. She observes that from a distance of 20 m from the tree, the angle of elevation to the top of the tree is 32°. What is the height of the tree to the nearest tenth of a metre?

2. From a lighthouse 16 m above sea level, the angle of depression to a small boat is 12°. How far from the foot of the lighthouse is the boat to the nearest metre?

3. A kite is 40 m high when 210 m of string is used. What angle does the kite string make with the horizontal to the nearest degree?

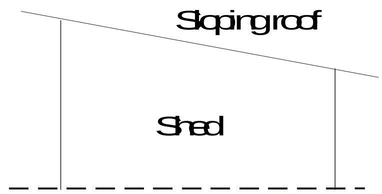
4. Marianne wants to know how far it is across a river. She notices a tree at point X straight across from point Y. She walks 10 m along the river bank to point Z and observes that the angle to the tree is 65°. What is the distance across the river from point X to point Y to the nearest hundredth of a metre?



5. A plot of land has the shape of a right triangle. The longest side is 37 m and lies at an angle of 53° to the shortest side. Find the area of the plot to the nearest square metre.

6. A painter is using a ladder to paint a high factory wall. The ladder is 5 m long and for safety, must never be used at an angle to the vertical of less than 15° or more than 40°. If he can reach 1 m above the top of the ladder, what is the maximum height to the nearest tenth of a metre that he can paint? (Hint: what angle would give the painter the greatest height?)

7. Jack is building a garden shed of his own design. It has a simple sloping roof. The two walls on which the roof will rest are 3.2 m apart and one wall is 0.5 m higher than the other. Allowing 0.25 m for overhang at either end, how long (to the nearest hundredth) will the roof beams have to be? What will the slope of the roof be to the nearest degree with respect to the horizontal?



8. Joan is welding a piece of modern sculpture. Part of the design includes an A-frame structure. Joan wants the two thin bars that make up the sides of the frame to form an angle of 54° at the top and she wants the frame to be 2.2 m high. How long will each bar have to be to the nearest thousandth? (Hint: you need a right triangle to use a trigonometric ratio.)

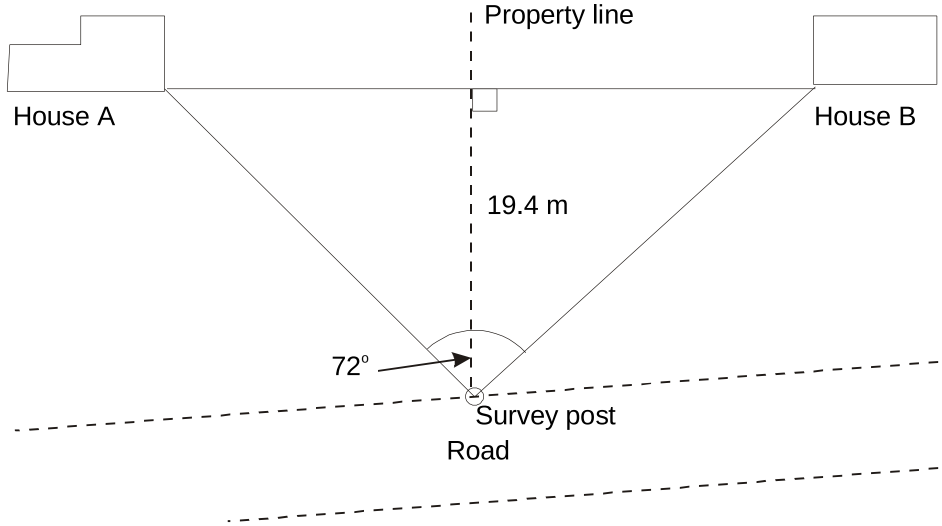
9. A new ski-lift is being built at the slopes. The base of the lift is at an elevation of 2 500 m, but the elevation of the top station is not accurately known. A survey of the site shows the base and the top station are 2 450 m apart in horizontal distance and a line of sight to the top station angles up at 38°. Find the length of steel cable (to the nearest 10 m) that will be needed for the endless loop on which the chairs will hang. Allow for an additional 5% of the total length for sags, joining, etc.

10. A mathematical spider spinning a web across an alleyway between two buildings manages to stretch a length of silken thread across from the top of a 2 m high doorway to a point on the wall of the opposite building 1 m above the ground. The spider just happened to make the angle of the thread exactly 25° to the horizontal. How far apart are the buildings to the nearest centimetre?

11. The roof of a small pup tent is made of a rectangular piece of material. If the tent is to be 2.2 m long, the roof sloping up at 48° to the horizontal and with the poles 1.4 m high, how many square metres of material will be needed to make the tent roof? (nearest hundredth)

Problem Set B

1. Given that the property line is halfway between the two houses on the plan below, what is the distance between the two houses to the nearest tenth of a metre?



2. How far is the corner of house B from the survey post to the nearest tenth of a metre?

3. A carpenter is instructed to cut a right-angled wooden wedge 25 cm long in the base with an angle of 12°. How long will the sloping surface of the wedge be to the nearest millimetre?

4. From a ladder, Wayne looks at a building 35 m away. He notes that the angle of elevation to the top of the building is 19° and the angle of depression to the bottom of the building is 7°. How high is the building?

5. A slide in the Water Park is 9 m high. If the actual length of the slide is 14 m, what angle does the slide make with the horizontal

Appendix A: Table of Trigonometric Ratios

Angle	sin	cos	tan	Angle	sin	cos	tan
0°	0.000	1.000	0.000	45°	0.707	0.707	1.000
1 °	0.017	1.000	0.017	46°	0.719	0.695	1.036
2 °	0.035	0.999	0.035	47°	0.731	0.682	1.072
3°	0.052	0.999	0.052	48°	0.743	0.669	1.111
4°	0.070	0.998	0.070	49°	0.755	0.656	1.150
5°	0.087	0.996	0.087	50°	0.766	0.643	1.192
6°	0.105	0.995	0.105	51°	0.777	0.629	1.235
7°	0.122	0.993	0.123	52°	0.788	0.616	1.280
8°	0.139	0.990	0.141	53°	0.799	0.602	1.327
9°	0.156	0.988	0.158	54°	0.809	0.588	1.376
10°	0.174	0.985	0.176	55°	0.819	0.574	1.428
11°	0.191	0.982	0.194	56°	0.829	0.559	1.483
12°	0.208	0.978	0.213	57°	0.839	0.545	1.540
13°	0.225	0.974	0.231	58°	0.848	0.535	1.600
14°	0.242	0.970	0.249	59°	0.857	0.515	1.664
15°	0.259	0.966	0.268	60°	0.866	0.500	1.732
16°	0.276	0.961	0.287	61°	0.875	0.485	1.804
17°	0.292	0.956	0.306	62°	0.883	0.469	1.881
18°	0.309	0.951	0.325	63°	0.891	0.454	1.963
19°	0.326	0.946	0.344	64°	0.899	0.438	2.050
20°	0.342	0.940	0.364	65°	0.906	0.423	2.145
21°	0.358	0.934	0.384	66°	0.914	0.407	2.246
22°	0.375	0.927	0.404	67°	0.921	0.391	2.356
23°	0.391	0.921	0.424	68°	0.927	0.375	2.475
24°	0.407	0.914	0.445	69°	0.934	0.358	2.605
25°	0.423	0.906	0.466	70°	0.940	0.342	2.747
26°	0.438	0.899	0.488	71°	0.946	0.326	2.904
27°	0.454	0.891	0.510	72°	0.951	0.309	3.078
28°	0.469	0.883	0.532	73°	0.956	0.292	3.271
29°	0.485	0.875	0.554	74°	0.961	0.276	3.487
30°	0.500	0.866	0.577	75°	0.966	0.259	3.732
31°	0.515	0.857	0.601	76°	0.970	0.242	4.011
32°	0.530	0.848	0.625	77°	0.974	0.225	4.332
33°	0.545	0.839	0.649	78°	0.978	0.208	4.705
34°	0.559	0.829	0.675	79°	0.982	0.191	5.145
35°	0.574	0.819	0.700	80°	0.985	0.174	5.671
36°	0.588	0.809	0.727	81°	0.988	0.156	6.314
37°	0.602	0.790	0.754	82°	0.990	0.139	7.115
38°	0.616	0.788	0.781	83°	0.993	0.122	8.144
39°	0.629	0.777	0.810	84°	0.995	0.105	9.514
40°	0.643	0.766	0.839	85°	0.996	0.087	11.430
41°	0.656	0.755	0.869	86°	0.998	0.070	14.301
42°	0.669	0.743	0.900	87°	0.999	0.052	19.081
43°	0.682	0.731	0.933	88°	0.999	0.035	28.636
44°	0.695	0.719	0.966	89°	1.000	0.017	57.290
45°	0.707	0.707	1.000	90°	1.000	0.000	

NOTE: values are correct to the nearest 1/1000th

Appendix B: Glossary

Acute angle

Any angle whose measure is greater than 0° and less than 90°. A right triangle always has two acute angles and one right angle. In Figure 22, ∠A and ∠B are acute angles.

Cosine of an angle

For a specified acute angle of a right triangle, the cosine of the angle is equal to the ratio of the length of the adjacent leg to the length of the hypotenuse. For the triangle in Figure 22:

$$\cos A = \frac{b}{c}$$

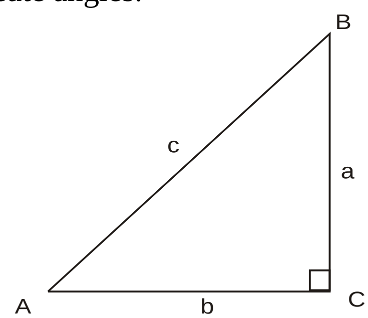


Figure 22

Hypotenuse

The longest side of a triangle, opposite the right angle. In Figure 22, side c is the hypotenuse.

Leg

Either of the two sides of a right triangle that are opposite the two acute angles. In Figure 22, sides b and a are the legs.

Pythagorean Theorem

The Pythagorean Theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the length of the legs. For the triangle in Figure 22:

$$c^2 = a^2 + b^2$$

Right triangle

A triangle that has a 90° angle. Figure 22 is a right triangle.

Sine of an angle

For a specified angle of a right angle triangle, the sine of the angle is equal to the ratio of the length of the opposite leg to the length of the hypotenuse. For the triangle in Figure 22:

$$\sin A = \frac{a}{c}$$

Tangent of an angle

For a specified angle in a right angle triangle, the tangent of an angle is equal to the ratio of the length of the opposite side to the length of the adjacent side. For the triangle in Figure 22:

$$\tan A = \frac{a}{b}$$

Answers

Exercise 5.1 - page 8

1. T	2. T	3. T	4. T	5. F	6. T
7. T	8. T	9. T	10. T		

Exercise 5.2 - pages 10-11

1. a. c or $\overline{AB}$ b. b or $\overline{AC}$ c. b or $\overline{AC}$ d. a or $\overline{BC}$ e. 50°
2. a. $\overline{PQ}$ or r b. $\overline{PR}$ or q c. $\overline{PR}$ or q d. $\overline{QR}$ or p e. $\overline{QR}$ or p
   f. $\overline{PQ}$ or r g. 70°
3. a. 60° b. 80° c. 15° d. 90° e. 36° f. 45°
4. a. large or L b. middle or M c. short or S d. yes

Exercise 5.3 - pages 14-16

1. a. $\angle C$ b. $\angle O$ c. $\angle B$ d. $\angle Y$
2. a. c b. $\overline{AB}$ c. $\overline{PR}$ d. e
3. a. 18.9 b. 34.0 c. 12.3d. 74.4 e. 21.2f. 1.1
   g. 13.2 h. 8.4
4. 65.6 cm
5. 3.8 m
6. 11.5 km
7. Yes, the distance between opposite corners is more than 2.4 m.

Exercise 5.4 - pages 19-20

1. calculator answers to 6 decimal places: tan 10° = 0.176327 tan 20° = 0.363970,
   tan 45° = 1.000000 tan 50° = 1.191754 tan 75° = 3.732051
2. a. 0.0000 b. 0.0175 c. 0.0875 d. 0.2679 e. 0.4040 f. 0.6104
   g. 1.0000 h. 1.2314 i. 2.0503 j. 0.0009 k. 14.3007 l. 57.2900
3. a. tan 25° = 0.5 b. tan 65° = 2.1
4. tan 88° = 28.636 tan 89° = 57.290 tan 89.9 = 572.957
   tan 89.99 = 5729.578 tan 90° undefined (your calculator should display ERROR)
5. tan 90° = undefined. Since it is impossible to draw a triangle with two 90° angles, tan 90° cannot be calculated.

Exercise 5.5 - pages 23-25

1. a. $\tan 15° = {}^{x}/_{10}$, x = 2.7 b. $\tan 30° = {}^{8}/_{x}$, x = 13.9 c. $\tan 65° = {}^{x}/_{19}$, x = 40.7
   d. $\tan 40° = {}^{x}/_{17.3}$, x = 14.5 e. $\tan 53° = {}^{239}/_{x}$, x = 180.1 f. $\tan 39° = {}^{x}/_{10.9}$, x = 8.8
   g. $\tan 80° = {}^{982}/_{x}$, x = 173.2 h. $\tan 13° = {}^{x}/_{50.7}$, x = 11.7
2. a. $\overline{BC} = 12.6$ b. $\overline{BC} = 9.1$
3. a. 34.4° b. 54° c. 40.6° d. 69°
4. $\angle A = 54.8°$ $\angle B = 35.2°$
5. $\angle B = 18°$ $\overline{AC} = 4.5$ $\overline{AB} = 14.7$
6. $\angle A = 40°$ $\overline{AC} = 88.2$ $\overline{AB} = 115.1$
7. 6.8°
8. 27.5 m + 1.6 m = 29.1 m
9. 31°

Exercise 5.6 - pages 28-31

1.

sin 1°	0.0175	cos 89°	0.0175
sin 5°	0.0872	cos 85°	0.0872
sin 10°	0.1736	cos 80°	0.1736
sin 22.5°	0.3827	cos 67.5°	0.3827
sin 30°	0.5000	cos 60°	0.5000
sin 45°	0.7071	cos 45°	0.7071
sin 60°	0.8660	cos 30°	0.8660
sin 67.5°	0.9239	cos 22.5°	0.9239
sin 80°	0.9848	cos 10°	0.9848
sin 85°	0.9962	cos 5°	0.9962
sin 89°	0.9998	cos 1°	0.9998

   Notice that both answer columns are the same. Also notice that sin 1° = cos 89° and that 1° + 89° = 90°, sin 5° = cos 85° and 5° + 85° = 90° and so on down the columns.
2. a. sin 37° = 3/5 = 0.6
   b. cos 53° = 3/5 = 0.6
   c. sin 53° = 4/5 = 0.8
   d. cos 37° = 4/5 = 0.8
   Notice that sine 37° = cos 53° since 37° + 53° = 90°. Also sin 53° = cos 37° because 53° + 37° = 90°.
   e. cos 70° = 0.342 since 20° + 70° = 90°
3. a. $\sin 30° = {}^{x}/_{40}$, x = 20 b. $\cos 25° = {}^{x}/_{13}$, x = 11.8 c. $\tan 60° = {}^{x}/_{9}$, x = 15.6
   d. $\sin 70° = {}^{142}/_{x}$,x = 151.1 e. $\cos 17° = {}^{91}/_{x}$, x = 95.2 f. $\sin 53° = {}^{x}/_{6.5}$, x = 5.2
   g. $\cos 45° = {}^{15.9}/_{x}$, x = 22.5 h. $\sin 72.5° = {}^{3.6}/_{x}$, x = 3.8
4. 184.4 cm
5. 23.1 cm
6. a. $\sin x = {}^{15}/_{20}$, $\angle x = 49°$ b. $\cos x = {}^{17.3}/_{30}$, $\angle x = 55°$ c. $\cos x = {}^{3.6}/_{5.1}$, $\angle x = 45°$
   d. $\sin x = {}^{305}/_{462}$, $\angle x = 41°$ e. $\cos x = {}^{4.85}/_{6.02}$, $\angle x = 36°$ f. $\cos x = {}^{83.4}/_{9.5,}$ $\angle x = 29°$
7. $\angle A = 42.5°$ and $\angle B = 47.5°$
8. 5.7°
9. 26.4 m
10. 345 km

Exercise 5.7 - pages 34-35

1. a. $\angle B = 50°$, $b = 6$, $c = 7.8$
   b. $\angle A = 63°$, $\angle B = 27°$, $b = 12.6$
   c. $\angle A = 25°$, $a = 6.8$, $b = 14.5$
   d. $c = 53.2$, $\angle A = 41°$, $\angle B = 49°$
   e. $\angle A = 48°$, $b = 54$, $c = 80.7$
   f. $\angle B = 67°$, $a = 3.2$, $c = 8.3$
2. $\angle B = 74°$, $a = 26.4$, $c = 95.7$
3. $b = 5$, $\angle A = 67°$, $\angle B = 23°$
4. $c = 1.4$, $\angle A = 45°$, $\angle B = 45°$
5. There are an infinite number of 40°-50°-90° triangles.
6. $\angle B = 30°$, $\overline{AC} = 13.9$, $\overline{BC} = 24$, $\overline{AB} = 27.7$
7. $\overline{BC} = 20 \sin 65° = 18.126$ and $\angle B = 25°$, therefore $\overline{CD} = 7.66$
8. 52.1 km north and 295.4 km east

Practice Test - pages 36-37

1. a. 0.5878 b. 0.9871 c. 19.0811 d. 0.0086
2. a. 9° b. 58° c. 33° d. 72°
3. a. 34.2 b. 6 c. 39.3 d. 54.3
   e. 51.2 f. 56.7
4. $\angle A = 50°$, $b = 73.8$, $c = 114.9$
5. $b = 18$, $\angle A = 53°$, $\angle B = 37°$
6. $\cos 25° = \dfrac{\overline{AC}}{1.2}$ so $\overline{AC} = 1.088$. Now $\sin 25° = \dfrac{\overline{CD}}{1.088}$ and $\overline{CD} = 0.46$
7. 2.8 km
8. 14 m

Problem Set A – pages 38-39

1. 12.5 m
2. 75 m
3. 11°
4. 21.45 m
5. 329 m^2
6. 5.8 m
7. roof beams are 3.73 m long, slope is 9°
8. 2.469 m
9. 6530 m
10. 214 cm
11. 8.27 m^2

Problem Set B – page 400

1. 28.2 m
2. 24.1 m
3. 256 mm
4. 16.3 m
5. 40°

Notes